

SOLUTION OF THE CAUCHY PROBLEM FOR THE NAVIER - STOKES AND EULER EQUATIONS

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Abstract. Some known results regarding the Euler and Navier-Stokes equations were obtained by different authors. Existence and smoothness of the Navier-Stokes solutions in two dimensions have been known for a long time. Leray [4] showed that the Navier-Stokes equations in three space dimensions have a weak solution. Scheffer [5], [6] and Shnirelman [7] obtained weak solution of the Euler equations with compact support in spacetime. Caffarelli-Kohn-Nirenberg [8] improved Scheffer's results, and F.-H. Lin [9] simplified the proof of the results of J. Leray. Many problems and conjectures about the behavior of solutions of the Euler and Navier-Stokes equations are described in the book of Bertozzi and Majda [1] or Constantin [2].

Solutions of the Navier-Stokes and Euler equations with initial conditions (Cauchy problem) for two and three dimensions are obtained in the convergence series form by the iterative method using the Fourier and Laplace transforms in this paper. For several combinations of problem parameters numerical results were obtained and presented as graphs.

1. The mathematical setup.

The Navier-Stokes equations describe the motion of a fluid in R^N ($N = 2$ or 3). We look for a viscous incompressible fluid filling all of R^N here. The Navier-Stokes equations are then given by

$$(1.1) \quad \frac{\partial u_k}{\partial t} + \sum_{n=1}^N u_n \frac{\partial u_k}{\partial x_n} = \nu \Delta u_k - \frac{\partial p}{\partial x_k} + f_k(x, t) \quad (x \in R^N, \quad t \geq 0, \quad 1 \leq k \leq N)$$

$$(1.2) \quad \operatorname{div} \vec{u} = \sum_{n=1}^N \frac{\partial u_n}{\partial x_n} = 0 \quad (x \in R^N, t \geq 0)$$

with initial conditions

$$(1.3) \quad \vec{u}(x, 0) = \vec{u}^0(x) \quad (x \in R^N)$$

Here $\vec{u}(x, t) = (u_k(x, t)) \in R^N$, ($1 \leq k \leq N$) – is an unknown velocity vector ($N = 2$ or 3), $p(x, t)$ – is an unknown pressure, $\vec{u}^0(x)$ is a given, C^∞ divergence-free vector field, $f_k(x, t)$ are components of a given, externally applied force $\vec{f}(x, t)$, ν is a positive coefficient of the viscosity (if $\nu = 0$ then (1.1) - (1.3) are the Euler equations), and $\Delta = \sum_{n=1}^N \frac{\partial^2}{\partial x_n^2}$ is the Laplacian in the space variables. Equation (1.1) is Newton's law for a fluid element subject. Equation (1.2) says that the fluid is incompressible. For physically reasonable solutions, we accept

$$(1.4) \quad u_k(x, t) \rightarrow 0, \quad \frac{\partial u_k}{\partial x_n} \rightarrow 0 \quad \text{as } |x| \rightarrow \infty \quad (1 \leq k \leq N, \quad 1 \leq n \leq N)$$

Hence, we will restrict attention to initial conditions \vec{u}^0 and force \vec{f} that satisfy

$$(1.5) \quad |\partial_x^\alpha \vec{u}^0(x)| \leq C_{\alpha K} (1 + |x|)^{-K} \quad \text{on } R^N \text{ for any } \alpha \text{ and } K > 0.$$

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and

$$(1.6) \quad | \partial_x^\alpha \partial_t^\beta \vec{f}(x, t) | \leq C_{\alpha\beta K} (1 + |x| + t)^{-K} \quad \text{on } \mathbb{R}^N \times [0, \infty) \text{ for any } \alpha, \beta \text{ and } K > 0.$$

We add $(-\sum_{n=1}^N u_n \frac{\partial u_k}{\partial x_n})$ to both sides of the equations (1.1). Then we have:

$$(1.7) \quad \frac{\partial u_k}{\partial t} = \nu \Delta u_k - \frac{\partial p}{\partial x_k} + f_k(x, t) - \sum_{n=1}^N u_n \frac{\partial u_k}{\partial x_n} \quad (x \in \mathbb{R}^N, \quad t \geq 0, \quad 1 \leq k \leq N)$$

$$(1.8) \quad \operatorname{div} \vec{u} = \sum_{n=1}^N \frac{\partial u_n}{\partial x_n} = 0 \quad (x \in \mathbb{R}^N, t \geq 0)$$

$$(1.9) \quad \vec{u}(x, 0) = \vec{u}^0(x) \quad (x \in \mathbb{R}^N)$$

$$(1.10) \quad u_k(x, t) \rightarrow 0, \quad \frac{\partial u_k}{\partial x_n} \rightarrow 0 \quad \text{as } |x| \rightarrow \infty \quad (1 \leq k \leq N, \quad 1 \leq n \leq N)$$

$$(1.11) \quad | \partial_x^\alpha \vec{u}^0(x) | \leq C_{\alpha K} (1 + |x|)^{-K} \quad \text{on } \mathbb{R}^N \text{ for any } \alpha \text{ and } K > 0.$$

$$(1.12) \quad | \partial_x^\alpha \partial_t^\beta \vec{f}(x, t) | \leq C_{\alpha\beta K} (1 + |x| + t)^{-K} \quad \text{on } \mathbb{R}^N \times [0, \infty) \text{ for any } \alpha, \beta \text{ and } K > 0.$$

We shall solve the system of equations (1.7) - (1.12) by the iterative method. To do so we write this system of equations in the following form:

$$(1.13) \quad \frac{\partial u_{jk}}{\partial t} = \nu \Delta u_{jk} - \frac{\partial p_j}{\partial x_k} + f_{jk}(x, t) \quad (x \in \mathbb{R}^N, \quad t \geq 0, \quad 1 \leq k \leq N)$$

$$(1.14) \quad \operatorname{div} \vec{u}_j = \sum_{n=1}^N \frac{\partial u_{jn}}{\partial x_n} = 0 \quad (x \in \mathbb{R}^N, t \geq 0)$$

$$(1.15) \quad \vec{u}_j(x, 0) = \vec{u}^0(x) \quad (x \in \mathbb{R}^N)$$

$$(1.16) \quad u_{jk}(x, t) \rightarrow 0, \quad \frac{\partial u_{jk}}{\partial x_n} \rightarrow 0 \quad \text{as } |x| \rightarrow \infty \quad (1 \leq k \leq N, \quad 1 \leq n \leq N)$$

$$(1.17) \quad | \partial_x^\alpha \vec{u}^0(x) | \leq C_{\alpha K} (1 + |x|)^{-K} \quad \text{on } \mathbb{R}^N \text{ for any } \alpha, \quad K > 0 \quad \text{and } C_{\alpha K} > 0.$$

$$(1.18) \quad |\partial_x^\alpha \partial_t^\beta \vec{f}(x, t)| \leq C_{\alpha\beta K} (1 + |x| + t)^{-K} \quad \text{on } \mathbb{R}^N \times [0, \infty) \text{ for any } \alpha, \beta, K > 0 \text{ and } C_{\alpha\beta K} > 0.$$

Here j is the number of the iterative process step ($j = 1, 2, 3, \dots$).

$$(1.19) \quad f_{jk}(x, t) = f_k(x, t) - \sum_{n=1}^N u_{j-1,n} \frac{\partial u_{j-1,k}}{\partial x_n} \quad (1 \leq k \leq N)$$

or the vector form

$$(1.20) \quad \vec{f}_j(x, t) = \vec{f}(x, t) - (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1}$$

For the first step of the iterative process ($j = 1$) we have:

$$(\vec{u}_0 \cdot \nabla) \vec{u}_0 = 0$$

and

$$\vec{f}_1(x, t) = \vec{f}(x, t)$$

2. Solution. Case $N = 2$.

We use Fourier transform (A.2) for equations (1.13) – (1.20) and get:

$$U_{jk}(\gamma_1, \gamma_2, t) = F[u_{jk}(x_1, x_2, t)]$$

$$F\left[\frac{\partial^2 u_{jk}(x_1, x_2, t)}{\partial x_s^2}\right] = -\gamma_s^2 U_{jk}(\gamma_1, \gamma_2, t) \quad [\text{use(1.16)}]$$

$$U_k^0(\gamma_1, \gamma_2) = F[u_k^0(x_1, x_2)]$$

$$P_j(\gamma_1, \gamma_2, t) = F[p_j(x_1, x_2, t)]$$

$$F_{jk}(\gamma_1, \gamma_2, t) = F[f_{jk}(x_1, x_2, t)]$$

$$k, s = 1, 2$$

and then:

$$(2.1) \quad \frac{\partial U_{j1}(\gamma_1, \gamma_2, t)}{\partial t} = -\nu(\gamma_1^2 + \gamma_2^2)U_{j1}(\gamma_1, \gamma_2, t) + i\gamma_1 P_j(\gamma_1, \gamma_2, t) + F_{j1}(\gamma_1, \gamma_2, t)$$

$$(2.2) \quad \frac{\partial U_{j2}(\gamma_1, \gamma_2, t)}{\partial t} = -\nu(\gamma_1^2 + \gamma_2^2)U_{j2}(\gamma_1, \gamma_2, t) + i\gamma_2 P_j(\gamma_1, \gamma_2, t) + F_{j2}(\gamma_1, \gamma_2, t)$$

$$(2.3) \quad \gamma_1 U_{j1}(\gamma_1, \gamma_2, t) + \gamma_2 U_{j2}(\gamma_1, \gamma_2, t) = 0$$

$$(2.4) \quad U_{j1}(\gamma_1, \gamma_2, 0) = U_1^0(\gamma_1, \gamma_2)$$

$$(2.5) \quad U_{j2}(\gamma_1, \gamma_2, 0) = U_2^0(\gamma_1, \gamma_2)$$

Hence eliminate $P_j(\gamma_1, \gamma_2, t)$ from equations (2.1), (2.2) and find:

$$(2.6) \quad \begin{aligned} & \frac{\partial}{\partial t} [U_{j2}(\gamma_1, \gamma_2, t) - \frac{\gamma_2}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, t)] = \\ & -\nu(\gamma_1^2 + \gamma_2^2) [U_{j2}(\gamma_1, \gamma_2, t) - \frac{\gamma_2}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, t)] + [F_{j2}(\gamma_1, \gamma_2, t) - \frac{\gamma_2}{\gamma_1} F_{j1}(\gamma_1, \gamma_2, t)] \end{aligned}$$

$$(2.7) \quad \gamma_1 U_{j1}(\gamma_1, \gamma_2, t) + \gamma_2 U_{j2}(\gamma_1, \gamma_2, t) = 0$$

$$(2.8) \quad U_{j1}(\gamma_1, \gamma_2, 0) = U_1^0(\gamma_1, \gamma_2)$$

$$(2.9) \quad U_{j2}(\gamma_1, \gamma_2, 0) = U_2^0(\gamma_1, \gamma_2)$$

We use Laplace transform (A.4), (A.5) for equations (2.6), (2.7) and have:

$$U_{jk}^\otimes(\gamma_1, \gamma_2, \eta) = L[U_{jk}(\gamma_1, \gamma_2, t)] \quad k = 1, 2$$

$$(2.10) \quad \begin{aligned} & \eta [U_{j2}^\otimes(\gamma_1, \gamma_2, \eta) - \frac{\gamma_2}{\gamma_1} U_{j1}^\otimes(\gamma_1, \gamma_2, \eta)] - [U_{j2}(\gamma_1, \gamma_2, 0) - \frac{\gamma_2}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, 0)] = \\ & -\nu(\gamma_1^2 + \gamma_2^2) [U_{j2}^\otimes(\gamma_1, \gamma_2, \eta) - \frac{\gamma_2}{\gamma_1} U_{j1}^\otimes(\gamma_1, \gamma_2, \eta)] + [F_{j2}^\otimes(\gamma_1, \gamma_2, \eta) - \frac{\gamma_2}{\gamma_1} F_{j1}^\otimes(\gamma_1, \gamma_2, \eta)] \end{aligned}$$

$$(2.11) \quad \gamma_1 U_{j1}^\otimes(\gamma_1, \gamma_2, \eta) + \gamma_2 U_{j2}^\otimes(\gamma_1, \gamma_2, \eta) = 0$$

$$(2.12) \quad U_{j1}(\gamma_1, \gamma_2, 0) = U_1^0(\gamma_1, \gamma_2)$$

$$(2.13) \quad U_{j2}(\gamma_1, \gamma_2, 0) = U_2^0(\gamma_1, \gamma_2)$$

The solution of the system of equations (2.10) – (2.13) is:

$$(2.14) \quad U_{j1}^{\otimes}(\gamma_1, \gamma_2, \eta) = \frac{[\gamma_2^2 F_{j1}^{\otimes}(\gamma_1, \gamma_2, \eta) - \gamma_1 \gamma_2 F_{j2}^{\otimes}(\gamma_1, \gamma_2, \eta) + \gamma_2^2 U_1^0(\gamma_1, \gamma_2) - \gamma_1 \gamma_2 U_2^0(\gamma_1, \gamma_2)]}{(\gamma_1^2 + \gamma_2^2)[\eta + \nu(\gamma_1^2 + \gamma_2^2)]}$$

$$(2.15) \quad U_{j2}^{\otimes}(\gamma_1, \gamma_2, \eta) = \frac{[\gamma_1^2 F_{j2}^{\otimes}(\gamma_1, \gamma_2, \eta) - \gamma_1 \gamma_2 F_{j1}^{\otimes}(\gamma_1, \gamma_2, \eta) + \gamma_1^2 U_2^0(\gamma_1, \gamma_2) - \gamma_1 \gamma_2 U_1^0(\gamma_1, \gamma_2)]}{(\gamma_1^2 + \gamma_2^2)[\eta + \nu(\gamma_1^2 + \gamma_2^2)]}$$

Then we use the convolution formula (A.6) and integral (A.7) for (2.14), (2.15) and obtain:

$$(2.16) \quad \begin{aligned} U_{j1}(\gamma_1, \gamma_2, t) = & \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \frac{[\gamma_2^2 F_{j1}(\gamma_1, \gamma_2, \tau) - \gamma_1 \gamma_2 F_{j2}(\gamma_1, \gamma_2, \tau)]}{(\gamma_1^2 + \gamma_2^2)} d\tau + \\ & + e^{-\nu(\gamma_1^2 + \gamma_2^2)t} U_1^0(\gamma_1, \gamma_2) \end{aligned}$$

$$(2.17) \quad \begin{aligned} U_{j2}(\gamma_1, \gamma_2, t) = & \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \frac{[\gamma_1^2 F_{j2}(\gamma_1, \gamma_2, \tau) - \gamma_1 \gamma_2 F_{j1}(\gamma_1, \gamma_2, \tau)]}{(\gamma_1^2 + \gamma_2^2)} d\tau + \\ & + e^{-\nu(\gamma_1^2 + \gamma_2^2)t} U_2^0(\gamma_1, \gamma_2) \end{aligned}$$

$P_j(\gamma_1, \gamma_2, t)$ is obtained from (2.1) or (2.2) :

$$(2.18) \quad P_j(\gamma_1, \gamma_2, t) = i \frac{[\gamma_1 F_{j1}(\gamma_1, \gamma_2, t) + \gamma_2 F_{j2}(\gamma_1, \gamma_2, t)]}{(\gamma_1^2 + \gamma_2^2)}$$

Use of the Fourier inversion formula (A.2) and find:

$$\begin{aligned} u_{j1}(x_1, x_2, t) = & \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \frac{[\gamma_2^2 F_{j1}(\gamma_1, \gamma_2, \tau) - \gamma_1 \gamma_2 F_{j2}(\gamma_1, \gamma_2, \tau)]}{(\gamma_1^2 + \gamma_2^2)} d\tau + \right. \\ & \left. + e^{-\nu(\gamma_1^2 + \gamma_2^2)t} U_1^0(\gamma_1, \gamma_2) \right] e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 = \\ = & \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_2^2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j1}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 - \end{aligned}$$

$$\begin{aligned}
& - \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1 \gamma_2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j2}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\
& \quad \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 + \\
& + \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\nu(\gamma_1^2 + \gamma_2^2)t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} u_1^0(\tilde{x}_1, \tilde{x}_2) d\tilde{x}_1 d\tilde{x}_2 e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 = \\
(2.19) \quad & = S_{11}(f_{j1}) + S_{12}(f_{j2}) + B(u_1^0) \\
& u_{j2}(x_1, x_2, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \frac{[\gamma_1^2 F_{j2}(\gamma_1, \gamma_2, \tau) - \gamma_1 \gamma_2 F_{j1}(\gamma_1, \gamma_2, \tau)]}{(\gamma_1^2 + \gamma_2^2)} d\tau + \right. \\
& \quad \left. + e^{-\nu(\gamma_1^2 + \gamma_2^2)t} U_2^0(\gamma_1, \gamma_2) \right] e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 = \\
& = - \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1 \gamma_2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j1}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\
& \quad \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 + \\
& + \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1^2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j2}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\
& \quad \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 + \\
& + \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\nu(\gamma_1^2 + \gamma_2^2)t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} u_2^0(\tilde{x}_1, \tilde{x}_2) d\tilde{x}_1 d\tilde{x}_2 e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 = \\
(2.20) \quad & = S_{21}(f_{j1}) + S_{22}(f_{j2}) + B(u_2^0)
\end{aligned}$$

$$p_j(x_1, x_2, t) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{[\gamma_1 F_{j1}(\gamma_1, \gamma_2, t) + \gamma_2 F_{j2}(\gamma_1, \gamma_2, t)]}{(\gamma_1^2 + \gamma_2^2)} e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 =$$

$$\begin{aligned}
&= -\frac{i}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1}{(\gamma_1^2 + \gamma_2^2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j1}(\tilde{x}_1, \tilde{x}_2, t) d\tilde{x}_1 d\tilde{x}_2 e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 + \\
&\quad + \frac{i}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_2}{(\gamma_1^2 + \gamma_2^2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j2}(\tilde{x}_1, \tilde{x}_2, t) d\tilde{x}_1 d\tilde{x}_2 e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 = \\
(2.21) \quad &= \tilde{S}_1(f_{j1}) + \tilde{S}_2(f_{j2})
\end{aligned}$$

So, the integrals (2.19) – (2.21) exist by the restrictions (1.17) , (1.18) .
Here $S_{11}()$, $S_{12}()$, $S_{21}()$, $S_{22}()$, $B()$, $\tilde{S}_1()$, $\tilde{S}_2()$ are the integral - operators.

$$S_{12}() = S_{21}()$$

We have for the vector \vec{u}_j from the equations (2.19) – (2.20) :

$$(2.22) \quad \vec{u}_j = \bar{\bar{S}} \cdot \vec{f}_j + B(\vec{u}^0),$$

where $\bar{\bar{S}}$ is the matrix - operator:

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

We put \vec{f}_j from equation (1.20) into equation (2.22) and have:

$$\begin{aligned}
\vec{u}_j &= \bar{\bar{S}} \cdot (\vec{f} - (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1}) + B(\vec{u}^0) = \\
&= \bar{\bar{S}} \cdot \vec{f} - \bar{\bar{S}} \cdot (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1} + B(\vec{u}^0) = \\
(2.23) \quad &= \vec{u}_1 - \bar{\bar{S}} \cdot (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1}
\end{aligned}$$

Here \vec{u}_1 is the solution of the system of equations (1.13) – (1.20) with condition:

$$\sum_{n=1}^2 u_n \frac{\partial u_k}{\partial x_n} = 0 \quad k = 1, 2$$

For $j = 1$ formula (2.22) can be written as follows:

$$(2.24) \quad \vec{u}_1 = \bar{\bar{S}} \cdot \vec{f}_1 + B(\vec{u}^0), \quad \vec{f}_1(x, t) = \vec{f}(x, t)$$

If $t \rightarrow 0$ then $\vec{u}_1 \rightarrow \vec{u}^0$ (look at integral-operators $\bar{\bar{S}}, B()$ - integrals (2.19) , (2.20)).
For $j = 2$ we define from equation (1.20):

$$(2.25) \quad \vec{f}_2(x, t) = \vec{f}_1(x, t) - (\vec{u}_1 \cdot \nabla) \vec{u}_1$$

We denote:

$$(2.26) \quad \vec{f}_2^* = (\vec{u}_1 \cdot \nabla) \vec{u}_1$$

and then we have:

$$(2.27) \quad \vec{f}_2(x, t) = \vec{f}_1(x, t) - \vec{f}_2^*$$

Then we get \vec{u}_2 from (2.22), (2.24):

$$(2.28) \quad \vec{u}_2 = \bar{\bar{S}} \cdot \vec{f}_2 + B(\vec{u}^0) = \bar{\bar{S}} \cdot (\vec{f}_1 - \vec{f}_2^*) + B(\vec{u}^0) = \vec{u}_1 - \vec{u}_2^*$$

Here we have:

$$(2.29) \quad \vec{u}_2^* = \bar{\bar{S}} \cdot \vec{f}_2^*$$

If $t \rightarrow 0$ then $\vec{u}_2^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (2.19), (2.20)).
Continue for $j = 3$. We define from equation (1.20):

$$(2.30) \quad \vec{f}_3(x, t) = \vec{f}_1(x, t) - (\vec{u}_2 \cdot \nabla) \vec{u}_2$$

Here we have:

$$(2.31) \quad (\vec{u}_2 \cdot \nabla) \vec{u}_2 = ((\vec{u}_1 - \vec{u}_2^*) \cdot \nabla) (\vec{u}_1 - \vec{u}_2^*) = \vec{f}_2^* + \vec{f}_3^*$$

We denote in (2.31):

$$\vec{f}_3^* = -(\vec{u}_1 \cdot \nabla) \vec{u}_2^* - (\vec{u}_2^* \cdot \nabla) \vec{u}_1 + (\vec{u}_2^* \cdot \nabla) \vec{u}_2^*$$

and then we have:

$$(2.32) \quad \vec{f}_3(x, t) = \vec{f}_1(x, t) - \vec{f}_2^* - \vec{f}_3^*$$

Then we get \vec{u}_3 from (2.22), (2.24), (2.29):

$$(2.33) \quad \vec{u}_3 = \bar{\bar{S}} \cdot \vec{f}_3 + B(\vec{u}^0) = \bar{\bar{S}} \cdot (\vec{f}_1 - \vec{f}_2^* - \vec{f}_3^*) + B(\vec{u}^0) = \vec{u}_1 - \vec{u}_2^* - \vec{u}_3^*$$

Here we denote:

$$(2.34) \quad \vec{u}_3^* = \bar{\bar{S}} \cdot \vec{f}_3^*$$

If $t \rightarrow 0$ then $\vec{u}_3^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (2.19), (2.20)).
For $j = 4$. We define from equation (1.20):

$$(2.35) \quad \vec{f}_4(x, t) = \vec{f}_1(x, t) - (\vec{u}_3 \cdot \nabla) \vec{u}_3$$

Here we have:

$$(2.36) \quad (\vec{u}_3 \cdot \nabla) \vec{u}_3 = ((\vec{u}_2 - \vec{u}_3^*) \cdot \nabla) (\vec{u}_2 - \vec{u}_3^*) = \vec{f}_2^* + \vec{f}_3^* + \vec{f}_4^*$$

We denote in (2.36):

$$\vec{f}_4^* = -(\vec{u}_2 \cdot \nabla) \vec{u}_3^* - (\vec{u}_3^* \cdot \nabla) \vec{u}_2 + (\vec{u}_3^* \cdot \nabla) \vec{u}_3^*$$

and then we have:

$$(2.37) \quad \vec{f}_4(x, t) = \vec{f}_1(x, t) - \vec{f}_2^* - \vec{f}_3^* - \vec{f}_4^*$$

Then we get \vec{u}_4 from (2.22), (2.24), (2.29), (2.34):

$$(2.38) \quad \vec{u}_4 = \bar{\bar{S}} \cdot \vec{f}_4 + B(\vec{u}^0) = \bar{\bar{S}} \cdot (\vec{f}_1 - \vec{f}_2^* - \vec{f}_3^* - \vec{f}_4^*) + B(\vec{u}^0) = \vec{u}_1 - \vec{u}_2^* - \vec{u}_3^* - \vec{u}_4^*$$

Here we denote:

$$(2.39) \quad \vec{u}_4^* = \bar{\bar{S}} \cdot \vec{f}_4^*$$

If $t \rightarrow 0$ then $\vec{u}_4^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (2.19), (2.20)).

For arbitrary number j ($j \geq 2$). We define from equation (1.20):

$$(2.40) \quad \vec{f}_j(x, t) = \vec{f}_1(x, t) - (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1}$$

Here we have:

$$(2.41) \quad (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1} = \sum_{l=2}^j \vec{f}_l^*$$

and it follows:

$$(2.42) \quad \vec{f}_j = \vec{f}_1 - \sum_{l=2}^j \vec{f}_l^*$$

Then we get \vec{u}_j from (2.22), (2.24):

$$(2.43) \quad \vec{u}_j = \bar{\bar{S}} \cdot \vec{f}_j + B(\vec{u}^0) = \bar{\bar{S}} \cdot (\vec{f}_1 - \sum_{l=2}^j \vec{f}_l^*) + B(\vec{u}^0) = \vec{u}_1 - \sum_{l=2}^j \vec{u}_l^*$$

Here we denote:

$$(2.44) \quad \vec{u}_l^* = \bar{\bar{S}} \cdot \vec{f}_l^* \quad (2 \leq l \leq j)$$

If $t \rightarrow 0$ then $\vec{u}_l^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (2.19), (2.20)).

We consider the equations (2.24) - (2.44) and see that the series (2.43) converge for $j \rightarrow \infty$ with the conditions for the first step ($j = 1$) of the iterative process:

$$\sum_{n=1}^2 u_{0n} \frac{\partial u_{0k}}{\partial x_n} = 0 \quad k = 1, 2$$

and conditions

$$(2.45) \quad C_{\alpha K} \leq \frac{1}{2}, \quad C_{\alpha\beta K} \leq \frac{1}{2}$$

Here $C_{\alpha K}$ and $C_{\alpha\beta K}$ are received from (1.17), (1.18).

Hence, we receive from equation (2.23) when $j \rightarrow \infty$:

$$(2.46) \quad \vec{u}_\infty = \vec{u}_1 - \bar{\bar{S}} \cdot (\vec{u}_\infty \cdot \nabla) \vec{u}_\infty$$

Equation (2.46) describes the converging iterative process.

Then we have from formula (2.21) :

$$(2.47) \quad p_\infty = \tilde{S}_1(f_{\infty 1}) + \tilde{S}_2(f_{\infty 2})$$

Here $\vec{f}_\infty = (f_{\infty 1}, f_{\infty 2})$ is received from formula (2.42).

On the other hand we can transform the original system of differential equations (1.7) – (1.9) to the equivalent system of integral equations by the scheme of iterative process (2.22), (2.23) for vector \vec{u} :

$$(2.48) \quad \vec{u} = \vec{u}_1 - \bar{\bar{S}} \cdot (\vec{u} \cdot \nabla) \vec{u},$$

where \vec{u}_1 is from formula (2.24). We compare the equations (2.46) and (2.48) and see that the iterative process (2.46) converge to the solution of the system (2.48) and hence to the solution of the differential equations (1.7) – (1.9) with conditions (2.45).

In other words there exist smooth functions $p_\infty(\mathbf{x}, t)$, $u_{\infty i}(\mathbf{x}, t)$ ($i = 1, 2$) on $\mathbf{R}^2 \times [0, \infty)$ that satisfy (1.1), (1.2), (1.3) and

$$(2.49) \quad p_\infty, u_{\infty i} \in C^\infty(\mathbf{R}^2 \times [0, \infty)),$$

$$(2.50) \quad \int_{\mathbf{R}^2} |\vec{u}_\infty(\mathbf{x}, t)|^2 d\mathbf{x} < C$$

for all $t \geq 0$.

3. Solution. Case N = 3.

We use Fourier transform (A.3) for equations (1.13) – (1.20) and get:

$$U_{jk}(\gamma_1, \gamma_2, \gamma_3, t) = F[u_{jk}(x_1, x_2, x_3, t)]$$

$$F\left[\frac{\partial^2 u_{jk}(x_1, x_2, x_3, t)}{\partial x_s^2}\right] = -\gamma_s^2 U_{jk}(\gamma_1, \gamma_2, \gamma_3, t) \quad [\text{use(1.16)}]$$

$$U_k^0(\gamma_1, \gamma_2, \gamma_3) = F[u_k^0(x_1, x_2, x_3)]$$

$$P_j(\gamma_1, \gamma_2, \gamma_3, t) = F[p_j(x_1, x_2, x_3, t)]$$

$$F_{jk}(\gamma_1, \gamma_2, \gamma_3, t) = F[f_{jk}(x_1, x_2, x_3, t)]$$

$$k, s = 1, 2, 3$$

and then:

$$(3.1) \quad \frac{dU_{j1}(\gamma_1, \gamma_2, \gamma_3, t)}{dt} = -\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)U_{j1}(\gamma_1, \gamma_2, \gamma_3, t) + i\gamma_1 P_j(\gamma_1, \gamma_2, \gamma_3, t) + F_{j1}(\gamma_1, \gamma_2, \gamma_3, t)$$

$$(3.2) \quad \frac{dU_{j2}(\gamma_1, \gamma_2, \gamma_3, t)}{dt} = -\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)U_{j2}(\gamma_1, \gamma_2, \gamma_3, t) + i\gamma_2 P_j(\gamma_1, \gamma_2, \gamma_3, t) + F_{j2}(\gamma_1, \gamma_2, \gamma_3, t)$$

$$(3.3) \quad \frac{dU_{j3}(\gamma_1, \gamma_2, \gamma_3, t)}{dt} = -\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)U_{j3}(\gamma_1, \gamma_2, \gamma_3, t) + i\gamma_3 P_j(\gamma_1, \gamma_2, \gamma_3, t) + F_{j3}(\gamma_1, \gamma_2, \gamma_3, t)$$

$$(3.4) \quad \gamma_1 U_{j1}(\gamma_1, \gamma_2, \gamma_3, t) + \gamma_2 U_{j2}(\gamma_1, \gamma_2, \gamma_3, t) + \gamma_3 U_{j3}(\gamma_1, \gamma_2, \gamma_3, t) = 0$$

$$(3.5) \quad U_{j1}(\gamma_1, \gamma_2, \gamma_3, 0) = U_1^0(\gamma_1, \gamma_2, \gamma_3)$$

$$(3.6) \quad U_{j2}(\gamma_1, \gamma_2, \gamma_3, 0) = U_2^0(\gamma_1, \gamma_2, \gamma_3)$$

$$(3.7) \quad U_{j3}(\gamma_1, \gamma_2, \gamma_3, 0) = U_3^0(\gamma_1, \gamma_2, \gamma_3)$$

Hence eliminate $P_j(\gamma_1, \gamma_2, \gamma_3, t)$ from equations (3.1) – (3.3) and find:

$$\begin{aligned}
(3.8) \quad & \frac{d}{dt} [U_{j2}(\gamma_1, \gamma_2, \gamma_3, t) - \frac{\gamma_2}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, \gamma_3, t)] = -\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2) [U_{j2}(\gamma_1, \gamma_2, \gamma_3, t) - \\
& - \frac{\gamma_2}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, \gamma_3, t)] + [F_{j2}(\gamma_1, \gamma_2, \gamma_3, t) - \frac{\gamma_2}{\gamma_1} F_{j1}(\gamma_1, \gamma_2, \gamma_3, t)]
\end{aligned}$$

$$\begin{aligned}
(3.9) \quad & \frac{d}{dt} [U_{j3}(\gamma_1, \gamma_2, \gamma_3, t) - \frac{\gamma_3}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, \gamma_3, t)] = -\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2) [U_{j3}(\gamma_1, \gamma_2, \gamma_3, t) - \\
& - \frac{\gamma_3}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, \gamma_3, t)] + [F_{j3}(\gamma_1, \gamma_2, \gamma_3, t) - \frac{\gamma_3}{\gamma_1} F_{j1}(\gamma_1, \gamma_2, \gamma_3, t)]
\end{aligned}$$

$$(3.10) \quad \gamma_1 U_{j1}(\gamma_1, \gamma_2, \gamma_3, t) + \gamma_2 U_{j2}(\gamma_1, \gamma_2, \gamma_3, t) + \gamma_3 U_{j3}(\gamma_1, \gamma_2, \gamma_3, t) = 0$$

$$(3.11) \quad U_{j1}(\gamma_1, \gamma_2, \gamma_3, 0) = U_1^0(\gamma_1, \gamma_2, \gamma_3)$$

$$(3.12) \quad U_{j2}(\gamma_1, \gamma_2, \gamma_3, 0) = U_2^0(\gamma_1, \gamma_2, \gamma_3)$$

$$(3.13) \quad U_{j3}(\gamma_1, \gamma_2, \gamma_3, 0) = U_3^0(\gamma_1, \gamma_2, \gamma_3)$$

We use Laplace transform (A.4), (A.5) for equations (3.8) – (3.10) and have:

$$\begin{aligned}
& U_{jk}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) = L[U_{jk}(\gamma_1, \gamma_2, \gamma_3, t)] \quad k = 1, 2, 3 \\
(3.14) \quad & \eta [U_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \frac{\gamma_2}{\gamma_1} U_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)] - [U_{j2}(\gamma_1, \gamma_2, \gamma_3, 0) - \frac{\gamma_2}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, \gamma_3, 0)] = \\
& -\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2) [U_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \frac{\gamma_2}{\gamma_1} U_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)] + \\
& + [F_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \frac{\gamma_2}{\gamma_1} F_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)]
\end{aligned}$$

$$\begin{aligned}
(3.15) \quad & \eta [U_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \frac{\gamma_3}{\gamma_1} U_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)] - [U_{j3}(\gamma_1, \gamma_2, \gamma_3, 0) - \frac{\gamma_3}{\gamma_1} U_{j1}(\gamma_1, \gamma_2, \gamma_3, 0)] = \\
& -\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2) [U_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \frac{\gamma_3}{\gamma_1} U_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)] + \\
& + [F_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \frac{\gamma_3}{\gamma_1} F_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)]
\end{aligned}$$

$$(3.16) \quad \gamma_1 U_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) + \gamma_2 U_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) + \gamma_3 U_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) = 0$$

$$(3.17) \quad U_{j1}(\gamma_1, \gamma_2, \gamma_3, 0) = U_1^0(\gamma_1, \gamma_2, \gamma_3)$$

$$(3.18) \quad U_{j2}(\gamma_1, \gamma_2, \gamma_3, 0) = U_2^0(\gamma_1, \gamma_2, \gamma_3)$$

$$(3.19) \quad U_{j3}(\gamma_1, \gamma_2, \gamma_3, 0) = U_3^0(\gamma_1, \gamma_2, \gamma_3)$$

In the usual way the solution of the system of equations (3.14) – (3.16) with formulas (3.17) – (3.19) can be rewritten in the following form:

$$(3.20) \quad U_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) = \frac{[(\gamma_2^2 + \gamma_3^2)F_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \gamma_1\gamma_2 F_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \gamma_1\gamma_3 F_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)[\eta + \nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)]} + \frac{U_1^0(\gamma_1, \gamma_2, \gamma_3)}{[\eta + \nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)]}$$

$$(3.21) \quad U_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) = \frac{[(\gamma_3^2 + \gamma_1^2)F_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \gamma_2\gamma_3 F_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \gamma_2\gamma_1 F_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)[\eta + \nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)]} + \frac{U_2^0(\gamma_1, \gamma_2, \gamma_3)}{[\eta + \nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)]}$$

$$(3.22) \quad U_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) = \frac{[(\gamma_1^2 + \gamma_2^2)F_{j3}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \gamma_3\gamma_1 F_{j1}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta) - \gamma_3\gamma_2 F_{j2}^\otimes(\gamma_1, \gamma_2, \gamma_3, \eta)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)[\eta + \nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)]} + \frac{U_3^0(\gamma_1, \gamma_2, \gamma_3)}{[\eta + \nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)]}$$

Then we use the convolution formula (A.6) and integral (A.7) for (3.20) – (3.22) and obtain:

$$(3.23) \quad U_{j1}(\gamma_1, \gamma_2, \gamma_3, t) = \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[(\gamma_2^2 + \gamma_3^2)F_{j1}(\gamma_1, \gamma_2, \gamma_3, \tau) - \gamma_1\gamma_2 F_{j2}(\gamma_1, \gamma_2, \gamma_3, \tau) - \gamma_1\gamma_3 F_{j3}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau + e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} U_1^0(\gamma_1, \gamma_2, \gamma_3)$$

$$\begin{aligned}
& U_{j2}(\gamma_1, \gamma_2, \gamma_3, t) = \\
& \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[(\gamma_3^2 + \gamma_1^2)F_{j2}(\gamma_1, \gamma_2, \gamma_3, \tau) - \gamma_2\gamma_3F_{j3}(\gamma_1, \gamma_2, \gamma_3, \tau) - \gamma_2\gamma_1F_{j1}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau + \\
(3.24) \quad & + e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} U_2^0(\gamma_1, \gamma_2, \gamma_3)
\end{aligned}$$

$$\begin{aligned}
& U_{j3}(\gamma_1, \gamma_2, \gamma_3, t) = \\
& \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[(\gamma_1^2 + \gamma_2^2)F_{j3}(\gamma_1, \gamma_2, \gamma_3, \tau) - \gamma_3\gamma_1F_{j1}(\gamma_1, \gamma_2, \gamma_3, \tau) - \gamma_3\gamma_2F_{j2}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau + \\
(3.25) \quad & + e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} U_3^0(\gamma_1, \gamma_2, \gamma_3)
\end{aligned}$$

$P_j(\gamma_1, \gamma_2, \gamma_3, t)$ is obtained from (3.1) [(3.2) or (3.3)] :

$$(3.26) \quad P_j(\gamma_1, \gamma_2, \gamma_3, t) = i \frac{[\gamma_1 F_{j1}(\gamma_1, \gamma_2, \gamma_3, t) + \gamma_2 F_{j2}(\gamma_1, \gamma_2, \gamma_3, t) + \gamma_3 F_{j3}(\gamma_1, \gamma_2, \gamma_3, t)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)}$$

Use of the Fourier inversion formula (A.3) and find:

$$\begin{aligned}
u_{j1}(x_1, x_2, x_3, t) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[(\gamma_2^2 + \gamma_3^2)F_{j1}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau - \right. \\
& \quad \left. - \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[\gamma_1\gamma_2F_{j2}(\gamma_1, \gamma_2, \gamma_3, \tau) + \gamma_1\gamma_3F_{j3}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau + \right. \\
& \quad \left. + e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} U_1^0(\gamma_1, \gamma_2, \gamma_3) \right] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
&= \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(\gamma_2^2 + \gamma_3^2)}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right. \\
& \quad \left. \cdot f_{j1}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \right] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 - \\
& \quad - \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1\gamma_2}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right. \\
& \quad \left. \cdot f_{j2}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \right] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 - \\
& \quad - \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1\gamma_3}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right.
\end{aligned}$$

$$\begin{aligned}
& \cdot f_{j3}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \Big] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 + \\
& + \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right. \\
& \quad \cdot u_1^0(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 \Big] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
(3.27) \quad & = S_{11}(f_{j1}) + S_{12}(f_{j2}) + S_{13}(f_{j3}) + B(u_1^0)
\end{aligned}$$

$$\begin{aligned}
u_{j2}(x_1, x_2, x_3, t) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[(\gamma_3^2 + \gamma_1^2)F_{j2}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau - \right. \\
& \quad - \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[\gamma_2\gamma_3 F_{j3}(\gamma_1, \gamma_2, \gamma_3, \tau) + \gamma_2\gamma_1 F_{j1}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau + \\
& \quad \left. + e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} U_2^0(\gamma_1, \gamma_2, \gamma_3) \right] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
& = -\frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_2\gamma_1}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right. \\
& \quad \cdot f_{j1}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \Big] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 + \\
& + \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(\gamma_3^2 + \gamma_1^2)}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right. \\
& \quad \cdot f_{j2}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \Big] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 - \\
& - \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_2\gamma_3}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right. \\
& \quad \cdot f_{j3}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \Big] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 + \\
& + \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1\gamma_1 + \tilde{x}_2\gamma_2 + \tilde{x}_3\gamma_3)} \cdot \right. \\
& \quad \cdot u_2^0(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 \Big] e^{-i(x_1\gamma_1 + x_2\gamma_2 + x_3\gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
(3.28) \quad & = S_{21}(f_{j1}) + S_{22}(f_{j2}) + S_{23}(f_{j3}) + B(u_2^0)
\end{aligned}$$

$$u_{j3}(x_1, x_2, x_3, t) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[(\gamma_1^2 + \gamma_2^2)F_{j3}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau - \right.$$

$$\begin{aligned}
& - \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \frac{[\gamma_3 \gamma_1 F_{j1}(\gamma_1, \gamma_2, \gamma_3, \tau) + \gamma_3 \gamma_2 F_{j2}(\gamma_1, \gamma_2, \gamma_3, \tau)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} d\tau + \\
& \quad + e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} U_3^0(\gamma_1, \gamma_2, \gamma_3) \Big] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
& = - \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_3 \gamma_1}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2 + \tilde{x}_3 \gamma_3)} \cdot \right. \\
& \quad \cdot f_{j1}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \Big] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 - \\
& - \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_3 \gamma_2}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2 + \tilde{x}_3 \gamma_3)} \cdot \right. \\
& \quad \cdot f_{j2}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \Big] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 + \\
& + \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(\gamma_1^2 + \gamma_2^2)}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2 + \tilde{x}_3 \gamma_3)} \cdot \right. \\
& \quad \cdot f_{j3}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 d\tau \Big] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 + \\
& + \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\nu(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)t} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2 + \tilde{x}_3 \gamma_3)} \cdot \right. \\
& \quad \cdot u_3^0(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 \Big] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
(3.29) \quad & = S_{31}(f_{j1}) + S_{32}(f_{j2}) + S_{33}(f_{j3}) + B(u_3^0)
\end{aligned}$$

$$\begin{aligned}
p_j(x_1, x_2, x_3, t) &= \frac{i}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\frac{[\gamma_1 F_{j1}(\gamma_1, \gamma_2, \gamma_3, t) + \gamma_2 F_{j2}(\gamma_1, \gamma_2, \gamma_3, t)]}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} + \right. \\
& \quad \left. + \frac{\gamma_3 F_{j3}(\gamma_1, \gamma_2, \gamma_3, t)}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \right] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
&= \frac{i}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2 + \tilde{x}_3 \gamma_3)} \cdot \right. \\
& \quad \cdot f_{j1}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, t) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 \Big] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 + \\
& + \frac{i}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_2}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2 + \tilde{x}_3 \gamma_3)} \cdot \right. \\
& \quad \cdot f_{j2}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, t) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 \Big] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 +
\end{aligned}$$

$$\begin{aligned}
& + \frac{i}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_3}{(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2 + \tilde{x}_3 \gamma_3)} \cdot \right. \\
& \quad \left. \cdot f_{j3}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, t) d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 \right] e^{-i(x_1 \gamma_1 + x_2 \gamma_2 + x_3 \gamma_3)} d\gamma_1 d\gamma_2 d\gamma_3 = \\
(3.30) \quad & = \tilde{S}_1(f_{j1}) + \tilde{S}_2(f_{j2}) + \tilde{S}_3(f_{j3})
\end{aligned}$$

So, the integrals (3.27) – (3.30) exist by the restrictions (1.17) , (1.18) .

Here $S_{11}()$, $S_{12}()$, $S_{13}()$, $S_{21}()$, $S_{22}()$, $S_{23}()$, $S_{31}()$, $S_{32}()$, $S_{33}()$, $B()$, $\tilde{S}_1()$, $\tilde{S}_2()$, $\tilde{S}_3()$ are the integral - operators.

$$S_{12}() = S_{21}()$$

$$S_{13}() = S_{31}()$$

$$S_{23}() = S_{32}()$$

We have for the vector \vec{u}_j from the equations (3.27) – (3.29) :

$$(3.31) \quad \vec{u}_j = \bar{\bar{S}} \cdot \vec{f}_j + B(\vec{u}^0),$$

where $\bar{\bar{S}}$ is the matrix - operator:

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

We put \vec{f}_j from equation (1.20) into equation (3.31) and have:

$$\begin{aligned}
& \vec{u}_j = \bar{\bar{S}} \cdot (\vec{f} - (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1}) + B(\vec{u}^0) = \\
& = \bar{\bar{S}} \cdot \vec{f} - \bar{\bar{S}} \cdot (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1} + B(\vec{u}^0) = \\
(3.32) \quad & = \vec{u}_1 - \bar{\bar{S}} \cdot (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1}
\end{aligned}$$

Here \vec{u}_1 is the solution of the system of equations (1.13) – (1.20) with condition:

$$\sum_{n=1}^3 u_n \frac{\partial u_k}{\partial x_n} = 0 \quad k = 1, 2, 3$$

For $j = 1$ formula (3.31) can be written as follows:

$$(3.33) \quad \vec{u}_1 = \bar{\bar{S}} \cdot \vec{f}_1 + B(\vec{u}^0), \quad \vec{f}_1(x, t) = \vec{f}(x, t)$$

If $t \rightarrow 0$ then $\vec{u}_1 \rightarrow \vec{u}^0$ (look at integral-operators $\bar{\bar{S}}, B()$ - integrals (3.27) - (3.29)).
For $j = 2$ we define from equation (1.20):

$$(3.34) \quad \vec{f}_2(x, t) = \vec{f}_1(x, t) - (\vec{u}_1 \cdot \nabla) \vec{u}_1$$

We denote:

$$(3.35) \quad \vec{f}_2^* = (\vec{u}_1 \cdot \nabla) \vec{u}_1$$

and then we have:

$$(3.36) \quad \vec{f}_2(x, t) = \vec{f}_1(x, t) - \vec{f}_2^*$$

Then we get \vec{u}_2 from (3.31), (3.33):

$$(3.37) \quad \vec{u}_2 = \bar{\bar{S}} \cdot \vec{f}_2 + B(\vec{u}^0) = \bar{\bar{S}} \cdot (\vec{f}_1 - \vec{f}_2^*) + B(\vec{u}^0) = \vec{u}_1 - \vec{u}_2^*$$

Here we have:

$$(3.38) \quad \vec{u}_2^* = \bar{\bar{S}} \cdot \vec{f}_2^*$$

If $t \rightarrow 0$ then $\vec{u}_2^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (3.27) - (3.29)).
Continue for $j = 3$. We define from equation (1.20):

$$(3.39) \quad \vec{f}_3(x, t) = \vec{f}_1(x, t) - (\vec{u}_2 \cdot \nabla) \vec{u}_2$$

Here we have:

$$(3.40) \quad (\vec{u}_2 \cdot \nabla) \vec{u}_2 = ((\vec{u}_1 - \vec{u}_2^*) \cdot \nabla) (\vec{u}_1 - \vec{u}_2^*) = \vec{f}_2^* + \vec{f}_3^*$$

We denote in (3.40):

$$\vec{f}_3^* = -(\vec{u}_1 \cdot \nabla) \vec{u}_2^* - (\vec{u}_2^* \cdot \nabla) \vec{u}_1 + (\vec{u}_2^* \cdot \nabla) \vec{u}_2^*$$

and then we have:

$$(3.41) \quad \vec{f}_3(x, t) = \vec{f}_1(x, t) - \vec{f}_2^* - \vec{f}_3^*$$

Then we get \vec{u}_3 from (3.31), (3.33), (3.38):

$$(3.42) \quad \vec{u}_3 = \bar{\bar{S}} \cdot \vec{f}_3 + B(\vec{u}^0) = \bar{\bar{S}} \cdot (\vec{f}_1 - \vec{f}_2^* - \vec{f}_3^*) + B(\vec{u}^0) = \vec{u}_1 - \vec{u}_2^* - \vec{u}_3^*$$

Here we denote:

$$(3.43) \quad \vec{u}_3^* = \bar{\bar{S}} \cdot \vec{f}_3^*$$

If $t \rightarrow 0$ then $\vec{u}_3^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (3.27) - (3.29)).
For $j = 4$. We define from equation (1.20):

$$(3.44) \quad \vec{f}_4(x, t) = \vec{f}_1(x, t) - (\vec{u}_3 \cdot \nabla) \vec{u}_3$$

Here we have:

$$(3.45) \quad (\vec{u}_3 \cdot \nabla) \vec{u}_3 = ((\vec{u}_2 - \vec{u}_3^*) \cdot \nabla) (\vec{u}_2 - \vec{u}_3^*) = \vec{f}_2^* + \vec{f}_3^* + \vec{f}_4^*$$

We denote in (3.45):

$$\vec{f}_4^* = -(\vec{u}_2 \cdot \nabla) \vec{u}_3^* - (\vec{u}_3^* \cdot \nabla) \vec{u}_2 + (\vec{u}_3^* \cdot \nabla) \vec{u}_3^*$$

and then we have:

$$(3.46) \quad \vec{f}_4(x, t) = \vec{f}_1(x, t) - \vec{f}_2^* - \vec{f}_3^* - \vec{f}_4^*$$

Then we get \vec{u}_4 from (3.31), (3.33), (3.38), (3.43):

$$(3.47) \quad \vec{u}_4 = \bar{\bar{S}} \cdot (\vec{f}_1 - \vec{f}_2^* - \vec{f}_3^* - \vec{f}_4^*) + B(\vec{u}^0) = \vec{u}_1 - \vec{u}_2^* - \vec{u}_3^* - \vec{u}_4^*$$

Here we denote:

$$(3.48) \quad \vec{u}_4^* = \bar{\bar{S}} \cdot \vec{f}_4^*$$

If $t \rightarrow 0$ then $\vec{u}_4^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (3.27) - (3.29)).
For arbitrary number j ($j \geq 2$). We define from equation (1.20):

$$(3.49) \quad \vec{f}_j(x, t) = \vec{f}_1(x, t) - (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1}$$

Here we have:

$$(3.50) \quad (\vec{u}_{j-1} \cdot \nabla) \vec{u}_{j-1} = \sum_{l=2}^j \vec{f}_l^*$$

and it follows:

$$(3.51) \quad \vec{f}_j = \vec{f}_1 - \sum_{l=2}^j \vec{f}_l^*$$

Then we get \vec{u}_j from (3.31), (3.33)

$$(3.52) \quad \vec{u}_j = \bar{\bar{S}} \cdot \vec{f}_j + B(\vec{u}^0) = \bar{\bar{S}} \cdot (\vec{f}_1 - \sum_{l=2}^j \vec{f}_l^*) + B(\vec{u}^0) = \vec{u}_1 - \sum_{l=2}^j \vec{u}_l^*$$

Here we denote:

$$(3.53) \quad \vec{u}_l^* = \bar{\bar{S}} \cdot \vec{f}_l^* \quad (2 \leq l \leq j)$$

If $t \rightarrow 0$ then $\vec{u}_l^* \rightarrow 0$ (look at integral-operator $\bar{\bar{S}}$ - integrals (3.27), (3.29)).

We consider the equations (3.33) - (3.53) and see that the series (3.52) converge for $j \rightarrow \infty$ with the conditions for the first step ($j = 1$) of the iterative process:

$$\sum_{n=1}^3 u_{0n} \frac{\partial u_{0k}}{\partial x_n} = 0 \quad k = 1, 2, 3$$

and conditions

$$(3.54) \quad C_{\alpha K} \leq \frac{1}{2}, \quad C_{\alpha\beta K} \leq \frac{1}{2}.$$

Here $C_{\alpha K}$ and $C_{\alpha\beta K}$ are received from (1.17), (1.18).

Hence, we receive from equation (3.32) when $j \rightarrow \infty$:

$$(3.55) \quad \vec{u}_\infty = \vec{u}_1 - \bar{\bar{S}} \cdot (\vec{u}_\infty \cdot \nabla) \vec{u}_\infty$$

Equation (3.55) describes the converging iterative process.

Then we have from formula (3.30) :

$$(3.56) \quad p_\infty = \tilde{S}_1(f_{\infty 1}) + \tilde{S}_2(f_{\infty 2}) + \tilde{S}_3(f_{\infty 3})$$

Here $\vec{f}_\infty = (f_{\infty 1}, f_{\infty 2}, f_{\infty 3})$ is received from formula (3.51).

On the other hand we can transform the original system of differential equations (1.7) – (1.9) to the equivalent system of integral equations by the scheme of iterative process (3.31), (3.32) for vector \vec{u} :

$$(3.57) \quad \vec{u} = \vec{u}_1 - \bar{\bar{S}} \cdot (\vec{u} \cdot \nabla) \vec{u},$$

where \vec{u}_1 is from formula (3.33). We compare the equations (3.55) and (3.57) and see that the iterative process (3.55) converge to the solution of the system (3.57) and hence to the solution of the differential equations (1.7) – (1.9) with conditions (3.54).

In other words there exist smooth functions $\mathbf{p}_\infty(\mathbf{x}, t)$, $\mathbf{u}_{\infty i}(\mathbf{x}, t)$ ($i = 1, 2, 3$) on $\mathbf{R}^3 \times [0, \infty)$ that satisfy (1.1), (1.2), (1.3) and

$$(3.58) \quad \mathbf{p}_\infty, \mathbf{u}_{\infty i} \in \mathbf{C}^\infty(\mathbf{R}^3 \times [0, \infty)),$$

$$(3.59) \quad \int_{\mathbf{R}^3} |\vec{\mathbf{u}}_\infty(\mathbf{x}, \mathbf{t})|^2 d\mathbf{x} < \mathbf{C}$$

for all $\mathbf{t} \geq 0$.

In the following chapters 4 and 5 we describe in further details examples of the solutions for the Navier-Stokes and Euler problems with various applied forces and different values of the viscosity coefficient ν .

4. Example of the solution of the Cauchy problem for the Euler equations by the described iterative method with a particular applied force ($N = 2$).

We will consider an example of the solution of the Cauchy problem for the Euler equations ($\nu = 0$) for $N = 2$ and with initial conditions:

$$(4.1) \quad \vec{u}(x, 0) = \vec{u}^0(x) = 0 \quad (x \in R^2)$$

Hence, and from formulas (2.19), (2.20) for arbitrary step j of the iterative process, it follows:

$$(4.2) \quad \begin{aligned} u_{j1}(x_1, x_2, t) = & \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_2^2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j1}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 - \\ & - \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1 \gamma_2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j2}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 \end{aligned}$$

$$\begin{aligned} u_{j2}(x_1, x_2, t) = & -\frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1 \gamma_2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j1}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 + \\ & + \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1^2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j2}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \end{aligned}$$

$$(4.3) \quad \cdot e^{-i(x_1\gamma_1+x_2\gamma_2)} d\gamma_1 d\gamma_2$$

We convert the Cartesian coordinates to the polar coordinates by formulas:

$x_1 = r \cdot \cos\varphi$; $x_2 = r \cdot \sin\varphi$; $\gamma_1 = \rho \cdot \cos\psi$; $\gamma_2 = \rho \cdot \sin\psi$; $\tilde{x}_1 = \tilde{r} \cdot \cos\tilde{\varphi}$; $\tilde{x}_2 = \tilde{r} \cdot \sin\tilde{\varphi}$; and obtain from formulas (4.2), (4.3):

$$u_{j1}(r, \varphi, t) = \frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \sin^2\psi \int_0^t \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot$$

$$\cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi -$$

$$- \frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \sin\psi\cos\psi \int_0^t \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot$$

$$\cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi$$

(4.4)

$$u_{j2}(r, \varphi, t) = -\frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \sin\psi\cos\psi \int_0^t \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot$$

$$\cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi +$$

$$+ \frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \cos^2\psi \int_0^t \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot$$

$$\cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi$$

(4.5)

We have the applied force \vec{f}_j for arbitrary step j of the iterative process:

$$(4.6) \quad f_{j\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau) = f_{j\tilde{r}}(\tilde{r}) e^{in_j\tilde{\varphi}} f_{j\tau}(\tau) \quad , \quad f_{j\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau) \equiv 0$$

or

$$(4.7) \quad f_{j\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau) \equiv 0 \quad , \quad f_{j\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau) = f_{j\tilde{\varphi}}(\tilde{r}) e^{in_j \tilde{\varphi}} f_{j\tau}(\tau)$$

where $f_{j\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau)$, $f_{j\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau)$ – radial and tangential components of the applied force.
 n_j - separate circumferential mode, $n_j = 0, 1, 2, 3, \dots$

We take the radial and tangential components of the applied force (4.6), (4.7) with condition (1.18) .
 For the radial component of the applied force we use De Moivre's formulas (A.8) and have:

$$(4.8) \quad \begin{aligned} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) &= f_{j\tilde{r}}(\tilde{r}) e^{in_j \tilde{\varphi}} \cos \tilde{\varphi} f_{j\tau}(\tau) = \frac{1}{2} f_{j\tilde{r}}(\tilde{r}) (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) f_{j\tau}(\tau) \\ f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) &= f_{j\tilde{\varphi}}(\tilde{r}) e^{in_j \tilde{\varphi}} \sin \tilde{\varphi} f_{j\tau}(\tau) = \frac{i}{2} f_{j\tilde{\varphi}}(\tilde{r}) (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) f_{j\tau}(\tau) \end{aligned}$$

We put the applied force components (4.8) in formulas (4.4), (4.5) , change the order of integration and find:

$$(4.9) \quad \begin{aligned} u_{jr1}(r, \varphi, t) &= \frac{1}{8\pi^2} \left[\int_0^\infty \int_0^{2\pi} \sin^2 \psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \right. \\ &\quad \cdot e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi - \\ &\quad - i \int_0^\infty \int_0^{2\pi} \sin \psi \cos \psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \\ &\quad \cdot e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi \left. \right] \int_0^t f_{j\tau}(\tau) d\tau \end{aligned}$$

$$\begin{aligned} u_{jr2}(r, \varphi, t) &= \frac{1}{8\pi^2} \left[- \int_0^\infty \int_0^{2\pi} \sin \psi \cos \psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \right. \\ &\quad \cdot e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi + \\ &\quad + i \int_0^\infty \int_0^{2\pi} \cos^2 \psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \end{aligned}$$

$$(4.10) \quad \left[\cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi \right] \int_0^t f_{j\tau}(\tau) d\tau$$

We denote:

$$(4.11) \quad u_{jt}(t) = \int_0^t f_{j\tau}(\tau) d\tau$$

and from formulas (4.9), (4.10) it follows:

$$(4.12) \quad \begin{aligned} u_{jr1}(r, \varphi, t) &= u_{jr1}(r, \varphi) u_{jt}(t) \\ u_{jr2}(r, \varphi, t) &= u_{jr2}(r, \varphi) u_{jt}(t) \end{aligned}$$

where

$$(4.13) \quad \begin{aligned} u_{jr1}(r, \varphi) &= \frac{1}{8\pi^2} \left[\int_0^\infty \int_0^{2\pi} \sin^2\psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \right. \\ &\quad \cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi - \\ &\quad - i \int_0^\infty \int_0^{2\pi} \sin\psi \cos\psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \\ &\quad \left. \cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi \right] \\ u_{jr2}(r, \varphi) &= \frac{1}{8\pi^2} \left[- \int_0^\infty \int_0^{2\pi} \sin\psi \cos\psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \right. \\ &\quad \left. \cdot e^{-ir\rho\cos(\psi-\varphi)} \rho d\rho d\psi + \right. \end{aligned}$$

$$\begin{aligned}
& + i \int_0^\infty \int_0^{2\pi} \cos^2 \psi \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} \cdot \\
& \cdot e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi \Big]
\end{aligned}$$

(4.14)

Let us denote internal integrals in (4.13), (4.14) as $I_\pm(\tilde{r}, \rho, \psi)$:

$$(4.15) \quad I_\pm(\tilde{r}, \rho, \psi) = \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} \pm e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi}$$

We have two integrals here. Plus (+) is for the first part of each integral (4.13), (4.14) and minus (-) is for the second part.

We substitute $\tilde{\theta}$ for $\tilde{\varphi}$: $\tilde{\theta} = \tilde{\varphi} - \psi$, $d\tilde{\theta} = d\tilde{\varphi}$ and receive:

$$(4.16) \quad I_\pm(\tilde{r}, \rho, \psi) = e^{i(n_j-1)\psi} \int_{-\psi}^{2\pi-\psi} e^{i\tilde{r}\rho \cos \tilde{\theta} + i(n_j-1)\tilde{\theta}} d\tilde{\theta} \pm e^{i(n_j+1)\psi} \int_{-\psi}^{2\pi-\psi} e^{i\tilde{r}\rho \cos \tilde{\theta} + i(n_j+1)\tilde{\theta}} d\tilde{\theta}$$

Then we use the Bessel function's integral representation (A.9) and have:

$$(4.17) \quad I_\pm(\tilde{r}, \rho, \psi) = 2\pi i^{(n_j-1)} e^{i(n_j-1)\psi} J_{n_j-1}(\tilde{r}\rho) \pm 2\pi i^{(n_j+1)} e^{i(n_j+1)\psi} J_{n_j+1}(\tilde{r}\rho)$$

Put $I_\pm(\tilde{r}, \rho, \psi)$ from (4.17) in formulas (4.13), (4.14), change order of integration and obtain:

$$(4.18) \quad u_{jr1}(r, \varphi) = \frac{1}{8\pi^2} \int_0^\infty \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} \left[\sin^2 \psi I_+(\tilde{r}, \rho, \psi) - i \sin \psi \cos \psi I_-(\tilde{r}, \rho, \psi) \right] e^{-ir\rho \cos(\psi-\varphi)} d\psi \tilde{r} d\tilde{r} \rho d\rho$$

$$(4.19) \quad u_{jr2}(r, \varphi) = \frac{1}{8\pi^2} \int_0^\infty \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} \left[-\sin \psi \cos \psi I_+(\tilde{r}, \rho, \psi) + i \cos^2 \psi I_-(\tilde{r}, \rho, \psi) \right] e^{-ir\rho \cos(\psi-\varphi)} d\psi \tilde{r} d\tilde{r} \rho d\rho$$

Then we group parts in brackets of formulas (4.18), (4.19), use De Moivre's formulas (A.8) and the Bessel function's properties. And we get:

$$(4.20) \quad u_{jr1}(r, \varphi) = -\frac{n_j i^{n_j}}{2\pi} \int_0^\infty \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} \sin \psi e^{-ir\rho \cos(\psi-\varphi) + i n_j \psi} d\psi J_{n_j}(\tilde{r}\rho) d\tilde{r} d\rho$$

$$(4.21) \quad u_{jr2}(r, \varphi) = \frac{n_j i^{n_j}}{2\pi} \int_0^\infty \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^{2\pi} \cos \psi e^{-ir\rho \cos(\psi-\varphi) + i n_j \psi} d\psi J_{n_j}(\tilde{r}\rho) d\tilde{r} d\rho$$

We substitute θ for ψ : $\theta = \psi - \varphi$, $d\theta = d\psi$ in the internal integrals of formulas (4.20), (4.21), use De Moivre's formulas (A.8) and the Bessel function's integral representation (A.9) and have from formulas (4.20), (4.21):

$$(4.22) \quad u_{jr1}(r, \varphi) = \frac{n_j}{2} e^{in_j\varphi} \int_0^\infty \left[e^{i\varphi} J_{n_{j+1}}(r\rho) + e^{-i\varphi} J_{n_{j-1}}(r\rho) \right] \int_0^\infty f_{j\tilde{r}}(\tilde{r}) J_{n_j}(\tilde{r}\rho) d\tilde{r} d\rho$$

$$(4.23) \quad u_{jr2}(r, \varphi) = \frac{i n_j}{2} e^{in_j\varphi} \int_0^\infty \left[e^{i\varphi} J_{n_{j+1}}(r\rho) - e^{-i\varphi} J_{n_{j-1}}(r\rho) \right] \int_0^\infty f_{j\tilde{r}}(\tilde{r}) J_{n_j}(\tilde{r}\rho) d\tilde{r} d\rho$$

Let us denote:

$$(4.24) \quad R_{j,n_j-1,r}(r) = \int_0^\infty \int_0^\infty f_{j\tilde{r}}(\tilde{r}) J_{n_j}(\tilde{r}\rho) J_{n_j-1}(r\rho) d\tilde{r} d\rho$$

$$(4.25) \quad R_{j,n_j+1,r}(r) = \int_0^\infty \int_0^\infty f_{j\tilde{r}}(\tilde{r}) J_{n_j}(\tilde{r}\rho) J_{n_j+1}(r\rho) d\tilde{r} d\rho$$

Then we have from formulas (4.22), (4.23):

$$(4.26) \quad u_{jr1}(r, \varphi) = \frac{n_j}{2} [R_{j,n_j-1,r}(r) e^{i(n_j-1)\varphi} + R_{j,n_j+1,r}(r) e^{i(n_j+1)\varphi}]$$

$$(4.27) \quad u_{jr2}(r, \varphi) = \frac{i n_j}{2} [R_{j,n_j-1,r}(r) e^{i(n_j-1)\varphi} - R_{j,n_j+1,r}(r) e^{i(n_j+1)\varphi}]$$

Then if $n_j = 0$ it follows from (4.24), (4.25), (4.26), (4.27) that $u_{jr1}(r, \varphi) = u_{jr2}(r, \varphi) = 0$ and hence $u_1 = u_2 = 0$.

In the equations below we will consider $n_j \geq 1$.

We change the order of integration in formulas (4.24), (4.25) and obtain:

$$(4.28) \quad R_{j,n_j-1,r}(r) = \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^\infty J_{n_j}(\tilde{r}\rho) J_{n_j-1}(r\rho) d\rho d\tilde{r}$$

$$(4.29) \quad R_{j,n_j+1,r}(r) = \int_0^\infty f_{j\tilde{r}}(\tilde{r}) \int_0^\infty J_{n_j}(\tilde{r}\rho) J_{n_j+1}(r\rho) d\rho d\tilde{r}$$

Internal integrals in formulas (4.28), (4.29) are established by the discontinuous integral of Weber and Schafheitlin (A.10) [12]. Then we have from (4.28), (4.29):

$$(4.30) \quad R_{j,n_j-1,r}(r) = r^{n_j-1} \int_r^\infty \frac{f_{j\tilde{r}}(\tilde{r})}{\tilde{r}^{n_j}} d\tilde{r}$$

$$(4.31) \quad R_{j,n_j+1,r}(r) = \frac{1}{r^{n_j+1}} \int_0^r \tilde{r}^{n_j} f_{j\tilde{r}}(\tilde{r}) d\tilde{r}$$

Now we integrate the solution (4.4), (4.5) by the tangential component of the applied force (4.7) for ($n_j \geq 1$). Then we use De Moivre's formulas (A.8) and have:

$$(4.32) \quad \begin{aligned} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) &= -f_{j\tilde{\varphi}}(\tilde{r}) e^{in_j \tilde{\varphi}} \sin \tilde{\varphi} f_{j\tau}(\tau) = -\frac{i}{2} f_{j\tilde{\varphi}}(\tilde{r}) (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) f_{j\tau}(\tau) \\ f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) &= f_{j\tilde{\varphi}}(\tilde{r}) e^{in_j \tilde{\varphi}} \cos \tilde{\varphi} f_{j\tau}(\tau) = \frac{1}{2} f_{j\tilde{\varphi}}(\tilde{r}) (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) f_{j\tau}(\tau) \end{aligned}$$

Hence formulas (4.32) are the components f_{j1} and f_{j2} from the tangential component of the applied force (4.7), while formulas (4.8) are the components f_{j1} and f_{j2} from the radial component of the applied force (4.6).

Let us put (4.32) in formulas (4.4), (4.5) and do the operations as we did in (4.9) – (4.27) ($n_j \geq 1$). We consider that $f_{j\tilde{\varphi}}(\tilde{r}) \cdot f_{j\tau}(\tau)$ is restricted by condition (1.18) and get:

$$(4.33) \quad R_{j,n_j-1,\varphi}(r) = - \int_0^\infty \int_0^\infty (f_{j\tilde{\varphi}}(\tilde{r}) \tilde{r})'_{\tilde{r}} J_{n_j}(\tilde{r}\rho) J_{n_j-1}(r\rho) d\tilde{r} d\rho$$

$$(4.34) \quad R_{j,n_j+1,\varphi}(r) = - \int_0^\infty \int_0^\infty (f_{j\tilde{\varphi}}(\tilde{r}) \tilde{r})'_{\tilde{r}} J_{n_j}(\tilde{r}\rho) J_{n_j+1}(r\rho) d\tilde{r} d\rho,$$

Here $()'_{\tilde{r}} \equiv \frac{\partial}{\partial \tilde{r}}$. Hence we have:

$$(4.35) \quad u_{j\varphi 1}(r, \varphi) = -\frac{i}{2} [R_{j,n_j-1,\varphi}(r) e^{i(n_j-1)\varphi} + R_{j,n_j+1,\varphi}(r) e^{i(n_j+1)\varphi}]$$

$$(4.36) \quad u_{j\varphi 2}(r, \varphi) = \frac{1}{2} [R_{j,n_j-1,\varphi}(r) e^{i(n_j-1)\varphi} - R_{j,n_j+1,\varphi}(r) e^{i(n_j+1)\varphi}]$$

We change the order of integration in formulas (4.33), (4.34) and obtain:

$$(4.37) \quad R_{j,n_j-1,\varphi}(r) = - \int_0^\infty (f_{j\tilde{\varphi}}(\tilde{r}) \tilde{r})'_{\tilde{r}} \int_0^\infty J_{n_j}(\tilde{r}\rho) J_{n_j-1}(r\rho) d\rho d\tilde{r}$$

$$(4.38) \quad R_{j,n_j+1,\varphi}(r) = - \int_0^\infty (f_{j\tilde{\varphi}}(\tilde{r}) \tilde{r})'_{\tilde{r}} \int_0^\infty J_{n_j}(\tilde{r}\rho) J_{n_j+1}(r\rho) d\rho d\tilde{r}$$

Internal integrals in formulas (4.37), (4.38) are established by the discontinuous integral of Weber and Schafheitlin (A.10)[12]. Then we have from (4.37), (4.38):

$$(4.39) \quad R_{j,n_j-1,\varphi}(r) = -r^{n_j-1} \int_r^\infty (f_{j\tilde{\varphi}}(\tilde{r}) \tilde{r})'_{\tilde{r}} \frac{d\tilde{r}}{\tilde{r}^{n_j}}$$

$$(4.40) \quad R_{j,n_j+1,\varphi}(r) = -\frac{1}{r^{n_j+1}} \int_0^r (f_{j\tilde{\varphi}}(\tilde{r}) \tilde{r})'_{\tilde{r}} \tilde{r}^{n_j} d\tilde{r}$$

We have obtain formulas (4.8) - (4.40) for an arbitrary step j of the iterative process and the applied forces (4.6) or (4.7).

Now we investigate the first step ($j = 1$, $n_1 = n = 1, 2, 3, \dots$) of the iterative process with the particular radial component of the applied force $f_1(x, t)$ [look at (2.24)]:

$$f_{1\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau) = f_{1\tilde{r}}(\tilde{r}) e^{in\tilde{\varphi}} f_{1\tau}(\tau) \quad , \quad f_{1\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau) \equiv 0$$

$$(4.41) \quad f_{1\tilde{r}}(\tilde{r}) = F_n \tilde{r}^{n+1} e^{-\mu_n \tilde{r}} \quad , \quad f_{1\tau}(\tau) = e^{-\sigma_n \tau}$$

F_n, μ_n, σ_n - constants. $0 < F_n < \infty$, $1 < \mu_n < \infty$, $1 < \sigma_n < \infty$.

Let us put the applied force (4.41) in formulas (4.30), (4.31), integrate and then we have:

$$(4.42) \quad R_{1,n-1,r}(r) = r^{n-1} \int_r^\infty \frac{F_n \tilde{r}^{n+1} e^{-\mu_n \tilde{r}}}{\tilde{r}^n} d\tilde{r} = F_n r^{n-1} \int_r^\infty e^{-\mu_n \tilde{r}} \tilde{r} d\tilde{r} = \frac{F_n r^{n-1}}{\mu_n^2} \Gamma(2, \mu_n r)$$

$$R_{1,n+1,r}(r) = \frac{1}{r^{n+1}} \int_0^r F_n \tilde{r}^{2n+1} e^{-\mu_n \tilde{r}} d\tilde{r} = \frac{F_n}{r^{n+1} \mu_n^{2n+2}} \gamma(2n+2, \mu_n r)$$

$$(4.43)$$

$\Gamma(\alpha, x), \gamma(\alpha, x)$ are the incomplete gamma functions [11].

Hence, and from formulas (4.26), (4.27) it follows by $j = 1$:

$$(4.44) \quad u_{1r1}(r, \varphi) = \frac{n}{2} [R_{1,n-1,r}(r) e^{i(n-1)\varphi} + R_{1,n+1,r}(r) e^{i(n+1)\varphi}]$$

$$(4.45) \quad u_{1r2}(r, \varphi) = \frac{i n}{2} [R_{1,n-1,r}(r) e^{i(n-1)\varphi} - R_{1,n+1,r}(r) e^{i(n+1)\varphi}]$$

and from formula (4.11):

$$(4.46) \quad u_{1t}(t) = \int_0^t f_{1\tau}(\tau) d\tau = \int_0^t e^{-\sigma_n \tau} d\tau = \frac{1}{\sigma_n} \gamma(1, \sigma_n t)$$

And we have the velocity \vec{u}_1 [look at (2.24)] from formulas (4.12):

$$\begin{aligned} u_{1r1}(r, \varphi, t) &= u_{1r1}(r, \varphi) u_{1t}(t) \\ (4.47) \quad u_{1r2}(r, \varphi, t) &= u_{1r2}(r, \varphi) u_{1t}(t) \end{aligned}$$

We obtain the following equations by performing appropriate transformations:

$$(4.48) \quad u_{1r}(r, \varphi, t) = \frac{n}{2} [R_{1,n-1,r}(r) + R_{1,n+1,r}(r)] e^{in\varphi} u_{1t}(t)$$

$$(4.49) \quad u_{1\varphi}(r, \varphi, t) = \frac{i n}{2} [R_{1,n-1,r}(r) - R_{1,n+1,r}(r)] e^{in\varphi} u_{1t}(t)$$

$u_{1r}(r, \varphi, t)$, $u_{1\varphi}(r, \varphi, t)$ are the radial and tangential components of the velocity \vec{u}_1 .
We have from formula (4.46):

$$(4.50) \quad \lim_{t \rightarrow 0} u_{1t}(t) = 0$$

Hence, and from formulas (4.47) – (4.49):

$$(4.51) \quad \begin{aligned} \lim_{t \rightarrow 0} u_{1r1}(r, \varphi, t) &= 0; & \lim_{t \rightarrow 0} u_{1r2}(r, \varphi, t) &= 0; \\ \lim_{t \rightarrow 0} u_{1\varphi}(r, \varphi, t) &= 0; & \lim_{t \rightarrow 0} u_{1\varphi}(r, \varphi, t) &= 0; \end{aligned}$$

In other words the velocity \vec{u}_1 satisfies the initial conditions (4.1).

We use the asymptotic properties of the incomplete gamma functions $\Gamma(\alpha, x)$, $\gamma(\alpha, x)$ and from formulas (4.42) – (4.45), (4.48), (4.49) we have the velocity \vec{u}_1 satisfies conditions (1.16) (for $r \rightarrow \infty$).

Let us continue investigation for the second step ($j = 2$) of the iterative process.

Find $\vec{f}_2^*(r, \varphi, t) = \{f_{21}^*, f_{22}^*\}$ - the first correction of the particular radial applied force $f_1(x, t)$ (4.41).

We have for \vec{f}_2^* from formula (2.26):

$$(4.52) \quad f_{21}^* = u_{1r1} \frac{\partial u_{1r1}}{\partial x_1} + u_{1r2} \frac{\partial u_{1r1}}{\partial x_2}$$

$$(4.53) \quad f_{22}^* = u_{1r1} \frac{\partial u_{1r2}}{\partial x_1} + u_{1r2} \frac{\partial u_{1r2}}{\partial x_2}$$

where u_{1r1} , u_{1r2} are the components of \vec{u}_1 and were taken from formulas (4.47).

We have here:

$$\begin{aligned}
\frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_1} &= \frac{\partial u_{1r1}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_1} + \frac{\partial u_{1r1}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_1} \\
\frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_2} &= \frac{\partial u_{1r1}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_2} + \frac{\partial u_{1r1}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_2} \\
\frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_1} &= \frac{\partial u_{1r2}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_1} + \frac{\partial u_{1r2}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_1} \\
\frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_2} &= \frac{\partial u_{1r2}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_2} + \frac{\partial u_{1r2}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_2} \\
\frac{\partial r}{\partial x_1} &= \cos \varphi, \quad \frac{\partial \varphi}{\partial x_1} = -\frac{\sin \varphi}{r}, \quad \frac{\partial r}{\partial x_2} = \sin \varphi, \quad \frac{\partial \varphi}{\partial x_2} = \frac{\cos \varphi}{r}
\end{aligned}
\tag{4.54}$$

Hence, we use formulas (4.44) – (4.47) for $u_{1r1}(r, \varphi, t)$, $u_{1r2}(r, \varphi, t)$ and have from (4.54):

$$\begin{aligned}
\frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_1} &= \frac{n}{2} \left\{ [R'_{1,n-1,r}(r) e^{i(n-1)\varphi} + R'_{1,n+1,r}(r) e^{i(n+1)\varphi}] \cos \varphi + \right. \\
&+ i [(n-1)R_{1,n-1,r}(r) e^{i(n-1)\varphi} + (n+1)R_{1,n+1,r}(r) e^{i(n+1)\varphi}] \left(-\frac{\sin \varphi}{r} \right) \Big\} u_{1t}(t) \\
\frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_2} &= \frac{n}{2} \left\{ [R'_{1,n-1,r}(r) e^{i(n-1)\varphi} + R'_{1,n+1,r}(r) e^{i(n+1)\varphi}] \sin \varphi + \right. \\
&+ i [(n-1)R_{1,n-1,r}(r) e^{i(n-1)\varphi} + (n+1)R_{1,n+1,r}(r) e^{i(n+1)\varphi}] \frac{\cos \varphi}{r} \Big\} u_{1t}(t) \\
\frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_1} &= \frac{in}{2} \left\{ [R'_{1,n-1,r}(r) e^{i(n-1)\varphi} - R'_{1,n+1,r}(r) e^{i(n+1)\varphi}] \cos \varphi + \right. \\
&+ i [(n-1)R_{1,n-1,r}(r) e^{i(n-1)\varphi} - (n+1)R_{1,n+1,r}(r) e^{i(n+1)\varphi}] \left(-\frac{\sin \varphi}{r} \right) \Big\} u_{1t}(t) \\
\frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_2} &= \frac{in}{2} \left\{ [R'_{1,n-1,r}(r) e^{i(n-1)\varphi} - R'_{1,n+1,r}(r) e^{i(n+1)\varphi}] \sin \varphi + \right. \\
&+ i [(n-1)R_{1,n-1,r}(r) e^{i(n-1)\varphi} - (n+1)R_{1,n+1,r}(r) e^{i(n+1)\varphi}] \frac{\cos \varphi}{r} \Big\} u_{1t}(t)
\end{aligned}
\tag{4.55}$$

where

$$R'_{1,n-1,r}(r) = \frac{dR_{1,n-1,r}(r)}{dr} = F_n \left[\frac{(n-1)r^{n-2}}{\mu_n^2} \Gamma(2, \mu_n r) - r^n e^{-\mu_n r} \right]$$

$$(4.56) \quad R'_{1,n+1,r}(r) = \frac{dR_{1,n+1,r}(r)}{dr} = F_n \left[-\frac{(n+1)}{r^{n+2}\mu_n^{2n+2}} \gamma(2n+2, \mu_n r) + r^n e^{-\mu_n r} \right]$$

Let us put u_{1r1} , u_{1r2} , $\frac{\partial u_{1r1}}{\partial x_1}$, $\frac{\partial u_{1r1}}{\partial x_2}$, $\frac{\partial u_{1r2}}{\partial x_1}$, $\frac{\partial u_{1r2}}{\partial x_2}$ from formulas (4.47), (4.55) in formulas (4.52), (4.53) for f_{21}^* , f_{22}^* .

After completing appropriate operations we have:

$$(4.57) \quad f_{21}^*(r, \varphi, t) = \frac{n^2}{2^2} [T_{2,2n-1,r}(r) e^{i(2n-1)\varphi} + T_{2,2n+1,r}(r) e^{i(2n+1)\varphi}] T_{2n}(t)$$

$$(4.58) \quad f_{22}^*(r, \varphi, t) = \frac{in^2}{2^2} [T_{2,2n-1,r}(r) e^{i(2n-1)\varphi} - T_{2,2n+1,r}(r) e^{i(2n+1)\varphi}] T_{2n}(t)$$

where

$$(4.59) \quad \begin{aligned} T_{2,2n-1,r}(r) &= [R_{1,n-1,r}(r) + R_{1,n+1,r}(r)] R'_{1,n-1,r}(r) - \frac{(n-1)R_{1,n-1,r}(r)}{r} [R_{1,n-1,r}(r) - R_{1,n+1,r}(r)] \\ T_{2,2n+1,r}(r) &= [R_{1,n-1,r}(r) + R_{1,n+1,r}(r)] R'_{1,n+1,r}(r) - \frac{(n+1)R_{1,n+1,r}(r)}{r} [R_{1,n-1,r}(r) - R_{1,n+1,r}(r)] \end{aligned}$$

$$(4.60) \quad T_{2n}(t) = u_{1t}^2(t)$$

We use formulas (4.42), (4.43) for $R_{1,n-1,r}(r)$, $R_{1,n+1,r}(r)$ and (4.56) for $R'_{1,n-1,r}(r)$, $R'_{1,n+1,r}(r)$ then do appropriate operations for $T_{2,2n-1,r}(r)$, $T_{2,2n+1,r}(r)$ and get:

$$(4.61) \quad \begin{aligned} T_{2,2n-1,r}(r) &= F_n^2 \left[-\frac{r^{2n-1}}{\mu_n^2} e^{-\mu_n r} \Gamma(2, \mu_n r) + \frac{2(n-1)}{r^3 \mu_n^{2n+4}} \gamma(2n+2, \mu_n r) \Gamma(2, \mu_n r) - \right. \\ &\quad \left. - \frac{1}{r \mu_n^{2n+2}} e^{-\mu_n r} \gamma(2n+2, \mu_n r) \right] \\ T_{2,2n+1,r}(r) &= F_n^2 \left[\frac{r^{2n-1}}{\mu_n^2} e^{-\mu_n r} \Gamma(2, \mu_n r) - \frac{2(n+1)}{r^3 \mu_n^{2n+4}} \gamma(2n+2, \mu_n r) \Gamma(2, \mu_n r) + \right. \\ &\quad \left. + \frac{1}{r \mu_n^{2n+2}} e^{-\mu_n r} \gamma(2n+2, \mu_n r) \right] \end{aligned}$$

For radial f_{2r}^* and tangential $f_{2\varphi}^*$ components of the first correction $\vec{f}_2^*(r, \varphi, t)$ of the particular radial applied force we have:

$$(4.62) \quad f_{2r}^*(r, \varphi, t) = \frac{n^2}{2^2} [T_{2,2n-1,r}(r) + T_{2,2n+1,r}(r)] e^{i 2n\varphi} T_{2n}(t) = \frac{n^2}{2^2} T_{2,2n,r}(r) e^{i 2n\varphi} T_{2n}(t)$$

$$(4.63) \quad f_{2\varphi}^*(r, \varphi, t) = \frac{in^2}{2^2} [T_{2,2n-1,r}(r) - T_{2,2n+1,r}(r)] e^{i 2n\varphi} T_{2n}(t) = \frac{in^2}{2^2} T_{2,2n,\varphi}(r) e^{i 2n\varphi} T_{2n}(t)$$

Here

$$(4.64) \quad T_{2,2n,r}(r) = -F_n^2 \frac{4}{r^3 \mu_n^{2n+4}} \gamma(2n+2, \mu_n r) \Gamma(2, \mu_n r)$$

$$T_{2,2n,\varphi}(r) = -F_n^2 \left[\frac{2r^{2n-1}}{\mu_n^2} e^{-\mu_n r} \Gamma(2, \mu_n r) - \frac{4n}{r^3 \mu_n^{2n+4}} \gamma(2n+2, \mu_n r) \Gamma(2, \mu_n r) + \frac{2}{r \mu_n^{2n+2}} e^{-\mu_n r} \gamma(2n+2, \mu_n r) \right]$$

(4.65)

We compare the particular radial applied force \vec{f}_1^* from (4.41) with the first correction \vec{f}_2^* from ((4.62) – (4.65)) of this particular radial applied force, and we have:

$$(4.66) \quad |\vec{f}_2^*| \ll |\vec{f}_1^*|$$

with condition

$$(4.67) \quad F_n \leq \frac{1}{n}$$

After the first step of the iterative process ($j = 1$) we obtained the velocity \vec{u}_1^* [see (4.47)]. Now we will calculate \vec{u}_2^* - the first correction of the velocity \vec{u}_1^* . Solution of this problem has two stages. On the first stage we find the part of the first correction \vec{u}_{2r}^* , corresponding to the radial component of the first correction of applied force f_{2r}^* from (4.62):

$$(4.68) \quad f_{2r}^*(r, \varphi, t) = \frac{n^2}{2^2} T_{2,2n,r}(r) e^{i 2n\varphi} T_{2n}(t) \quad , \quad f_{2\varphi}^*(r, \varphi, t) \equiv 0$$

On the second stage we calculate the other part of the first correction $\vec{u}_{2\varphi}^*$, corresponding to the tangential component of the first correction of applied force $f_{2\varphi}^*$ from (4.63):

$$(4.69) \quad f_{2r}^*(r, \varphi, t) \equiv 0 \quad , \quad f_{2\varphi}^*(r, \varphi, t) = \frac{in^2}{2^2} T_{2,2n,\varphi}(r) e^{i 2n\varphi} T_{2n}(t)$$

In other words

$$(4.70) \quad \vec{u}_2^* = \vec{u}_{2r}^* + \vec{u}_{2\varphi}^*, \quad \vec{u}_{2r}^* = \{u_{2r1}^*, u_{2r2}^*\}, \quad \vec{u}_{2\varphi}^* = \{u_{2\varphi1}^*, u_{2\varphi2}^*\}.$$

First stage: we use formulas (4.26) , (4.27) for components $u_{jr1}(r, \varphi)$, $u_{jr2}(r, \varphi)$ and formulas (4.30) , (4.31) for $R_{j,n_j-1,r}(r)$, $R_{j,n_j+1,r}(r)$ for $j = 2$ and then formulas (4.62) , (4.64) for $f_{2r}^*(r, \varphi, t)$, $T_{2,2n,r}(r)$. We do appropriate operations and have:

$$(4.71) \quad u_{2r1}^*(r, \varphi) = n[R_{2,2n-1,r}(r) e^{i(2n-1)\varphi} + R_{2,2n+1,r}(r) e^{i(2n+1)\varphi}]$$

$$(4.72) \quad u_{2r2}^*(r, \varphi) = i n[R_{2,2n-1,r}(r) e^{i(2n-1)\varphi} - R_{2,2n+1,r}(r) e^{i(2n+1)\varphi}]$$

Here

$$(4.73) \quad R_{2,2n-1,r}(r) = \frac{n^2 r^{2n-1}}{2^2} \int_r^\infty \frac{T_{2,2n,r}(\tilde{r})}{\tilde{r}^{2n}} d\tilde{r}$$

$$(4.74) \quad R_{2,2n+1,r}(r) = \frac{n^2}{2^2 r^{2n+1}} \int_0^r \tilde{r}^{2n} T_{2,2n,r}(\tilde{r}) d\tilde{r}$$

Second stage: we use formulas (4.35) , (4.36) for components $u_{j\varphi1}(r, \varphi)$, $u_{j\varphi2}(r, \varphi)$ and formulas (4.39) , (4.40) for $R_{j,n_j-1,\varphi}(r)$, $R_{j,n_j+1,\varphi}(r)$ for $j = 2$ and then formulas (4.63) , (4.65) for $f_{2\varphi}^*(r, \varphi, t)$, $T_{2,2n,\varphi}(r)$. We do appropriate operations and have:

$$(4.75) \quad u_{2\varphi1}^*(r, \varphi) = -\frac{i}{2}[R_{2,2n-1,\varphi}(r) e^{i(2n-1)\varphi} + R_{2,2n+1,\varphi}(r) e^{i(2n+1)\varphi}]$$

$$(4.76) \quad u_{2\varphi2}^*(r, \varphi) = \frac{1}{2}[R_{2,2n-1,\varphi}(r) e^{i(2n-1)\varphi} - R_{2,2n+1,\varphi}(r) e^{i(2n+1)\varphi}]$$

Here:

$$(4.77) \quad R_{2,2n-1,\varphi}(r) = -\frac{in^2 r^{2n-1}}{2^2} \int_r^\infty \frac{(T_{2,2n,\varphi}(\tilde{r}) \cdot \tilde{r})'_{\tilde{r}}}{\tilde{r}^{2n}} d\tilde{r}$$

$$(4.78) \quad R_{2,2n+1,\varphi}(r) = -\frac{in^2}{2^2 r^{2n+1}} \int_0^r \tilde{r}^{2n} (T_{2,2n,\varphi}(\tilde{r}) \cdot \tilde{r})'_{\tilde{r}} d\tilde{r}$$

Then we have:

$$(4.79) \quad \begin{aligned} u_{21}^*(r, \varphi) &= u_{2r1}^*(r, \varphi) + u_{2\varphi1}^*(r, \varphi) = \\ &= [nR_{2,2n-1,r}(r) - \frac{i}{2}R_{2,2n-1,\varphi}(r)] e^{i(2n-1)\varphi} + [nR_{2,2n+1,r}(r) - \frac{i}{2}R_{2,2n+1,\varphi}(r)] e^{i(2n+1)\varphi} \end{aligned}$$

$$\begin{aligned}
u_{22}^*(r, \varphi) &= u_{2r2}^*(r, \varphi) + u_{2\varphi2}^*(r, \varphi) = \\
&= i \left[nR_{2,2n-1,r}(r) - \frac{i}{2}R_{2,2n-1,\varphi}(r) \right] e^{i(2n-1)\varphi} - i \left[nR_{2,2n+1,r}(r) - \frac{i}{2}R_{2,2n+1,\varphi}(r) \right] e^{i(2n+1)\varphi}
\end{aligned}
\tag{4.80}$$

From formula (4.11) for $j = 2$ and formula (4.60) we have:

$$u_{2t}(t) = \int_0^t T_{2n}(\tau) d\tau = \frac{1}{\sigma_n^2} \left[t - \frac{2}{\sigma_n} \gamma(1, \sigma_n t) + \frac{1}{2\sigma_n} \gamma(1, 2\sigma_n t) \right]
\tag{4.81}$$

Hence, and from equation (4.12) it follows:

$$\begin{aligned}
u_{21}^*(r, \varphi, t) &= u_{21}^*(r, \varphi) u_{2t}(t) \\
u_{22}^*(r, \varphi, t) &= u_{22}^*(r, \varphi) u_{2t}(t)
\end{aligned}
\tag{4.82}$$

After completing appropriate operations we have:

$$u_{2r}^*(r, \varphi, t) = \left\{ \left[nR_{2,2n-1,r}(r) - \frac{i}{2}R_{2,2n-1,\varphi}(r) \right] + \left[nR_{2,2n+1,r}(r) - \frac{i}{2}R_{2,2n+1,\varphi}(r) \right] \right\} e^{i2n\varphi} u_{2t}(t)
\tag{4.83}$$

$$u_{2\varphi}^*(r, \varphi, t) = i \left\{ \left[nR_{2,2n-1,r}(r) - \frac{i}{2}R_{2,2n-1,\varphi}(r) \right] - \left[nR_{2,2n+1,r}(r) - \frac{i}{2}R_{2,2n+1,\varphi}(r) \right] \right\} e^{i2n\varphi} u_{2t}(t)
\tag{4.84}$$

Here $u_{2r}^*(r, \varphi, t)$, $u_{2\varphi}^*(r, \varphi, t)$ are the radial and tangential components of the first correction \vec{u}_2^* of the velocity \vec{u}_1 and

$$\begin{aligned}
\left[nR_{2,2n-1,r}(r) - \frac{i}{2}R_{2,2n-1,\varphi}(r) \right] &= \frac{F_n^2 n^2 r^{2n-1}}{2^2} \left[-\frac{\Gamma(1, 2\mu_n r)}{2\mu_n^2} - \frac{\Gamma(2, 2\mu_n r)}{2^2 \mu_n^2} + \right. \\
&\quad \left. + n \sum_{l=0}^{\infty} \frac{\Gamma(l+2, 2\mu_n r)}{(2n+2)_{l+1} 2^{l+1} \mu_n^2} - \sum_{l=0}^{\infty} \frac{\Gamma(l+3, 2\mu_n r)}{(2n+2)_{l+1} 2^{l+3} \mu_n^2} \right]
\end{aligned}
\tag{4.85}$$

$$\left[nR_{2,2n+1,r}(r) - \frac{i}{2}R_{2,2n+1,\varphi}(r) \right] = \frac{F_n^2 n^2}{2^2 r^{2n+1}} \left[-\frac{n(2n+1)}{2^{4n-2} \mu_n^{4n+2}} \gamma(4n, 2\mu_n r) + \right.$$

$$(4.86) \quad + \frac{2n(2n+1)}{\mu_n^{2n+2}} \gamma(2n, \mu_n r) r^{2n} e^{-\mu_n r} \Big]$$

From formulas (4.82) or (4.83), (4.84) with properties of $u_{2t}(t)$ - (4.81) it follows:

$$(4.87) \quad \lim_{t \rightarrow 0} u_{21}^*(r, \varphi, t) = 0; \quad \lim_{t \rightarrow 0} u_{22}^*(r, \varphi, t) = 0;$$

$$(4.88) \quad \lim_{t \rightarrow 0} u_{2r}^*(r, \varphi, t) = 0; \quad \lim_{t \rightarrow 0} u_{2\varphi}^*(r, \varphi, t) = 0;$$

and we have the velocity $\vec{u}_2 = \vec{u}_1 - \vec{u}_2^*$ [look at (2.28)] satisfying the initial conditions (4.1). We use the asymptotic properties of the incomplete gamma functions $\Gamma(\alpha, x)$, $\gamma(\alpha, x)$ and from formulas (4.85), (4.86) we have: the first correction \vec{u}_2^* and therefore the velocity \vec{u}_2 satisfies conditions (1.16) (for $r \rightarrow \infty$).

Let us compare the solution (4.47) or (4.48), (4.49) for \vec{u}_1 of the first step of iterative process with the first correction (4.82) or (4.83), (4.84) for \vec{u}_2^* , which is received on the second step of iterative process. We see that

$$(4.89) \quad |\vec{u}_2^*| << |\vec{u}_1|$$

with conditions

$$(4.90) \quad F_n \leq \frac{1}{n} \\ t \leq \sigma_n$$

By continuing this iterative process we can obtain next parts $\vec{u}_3^*, \vec{u}_4^*, \dots$ of the converging series for \vec{u} . For arbitrary step j of the iterative process we have by using formula (2.43):

$$(4.91) \quad \vec{u}_j = \vec{u}_1 - \sum_{l=2}^j \vec{u}_l^*$$

and then:

$$(4.92) \quad \lim_{j \rightarrow \infty} \vec{u}_j = \vec{u}$$

where \vec{u} is the solution of the problem (1.1) – (1.6) for $\nu = 0$.

Below we provide numerical analysis of these results for the following values of problem's parameters:

Circumferential modes $n = 1, 2, 3, 4, 5$.

$\sigma_n = 10$.

$0 \leq t \leq 10$.

Results were obtained for functions $\vec{u}_1 - (4.47)$ or $(4.48), (4.49)$; $\vec{u}_2^* - (4.82)$ or $(4.83), (4.84)$ with calculations of the incomplete gamma functions [11].

$\vec{u}_2 = \vec{u}_1 - \vec{u}_2^*$ and is shown in FIG. 4.1 - 4.5. The vector field \vec{u}_2 at distances $r = 1, 2, 3, 5, 7$ is represented by the dotted curves in left diagrams. The comparison of $|\vec{u}_1|$ (dashed plots) and $|\vec{u}_2^*|$ (solid plots) in plane $\varphi = [0, \pi]$, at distances $0 \leq r \leq 50$ is represented in right diagrams. This comparison shows $|\vec{u}_2^*| \ll |\vec{u}_1|$ and is corresponding to the conclusion (4.89).

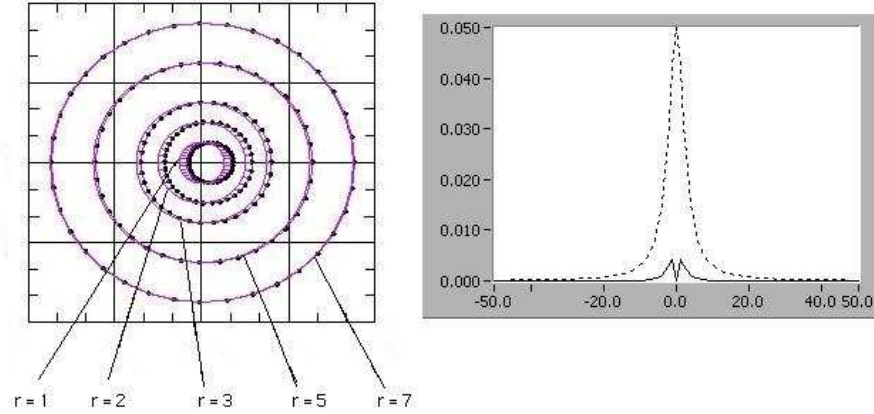


FIG.4.1. $n = 1, F_1 = 1, \mu_1 = 1$

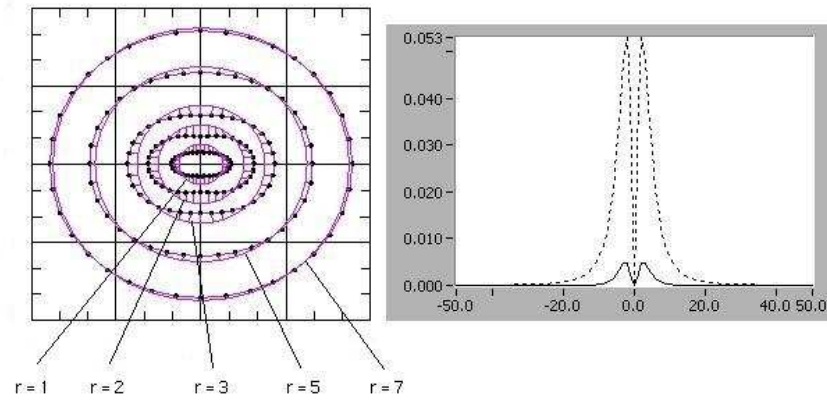
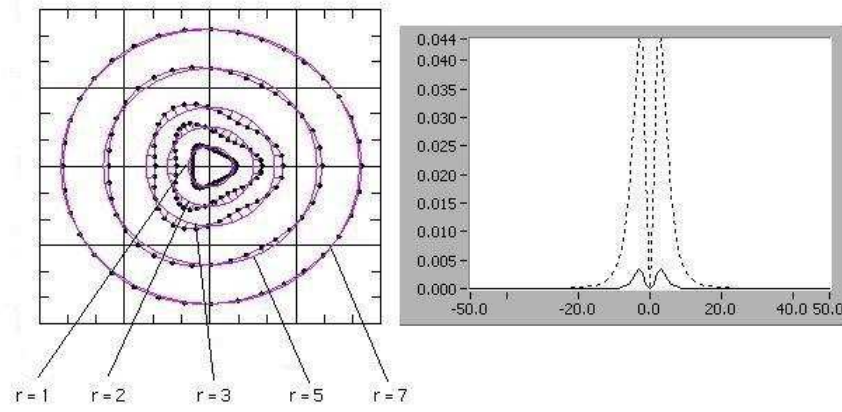
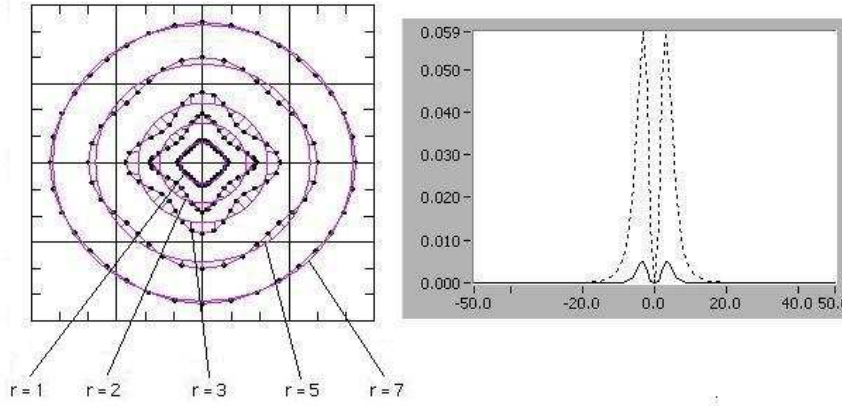
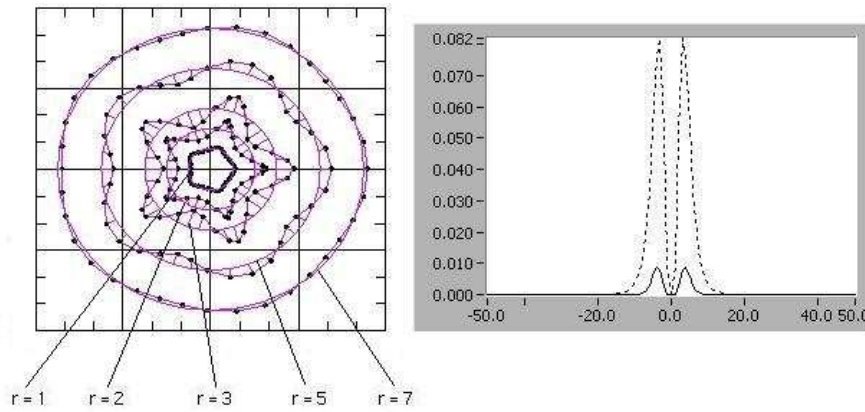


FIG.4.2. $n = 2, F_2 = 0.5, \mu_2 = 1$

FIG.4.3. $n = 3$, $F_3 = 0.33$, $\mu_3 = 1.3$ FIG.4.4. $n = 4$, $F_4 = 0.25$, $\mu_4 = 1.5$ FIG.4.5. $n = 5$, $F_5 = 0.2$, $\mu_5 = 1.7$

5. Example of the solution of the Cauchy problem for the Navier - Stokes equations by the described iterative method with a particular applied force ($N = 2$).

We will consider an example of the solution of the Cauchy problem for the Navier - Stokes equations for $N = 2$ and with initial conditions:

$$(5.1) \quad \vec{u}(x, 0) = \vec{u}^0(x) = 0 \quad (x \in R^2)$$

Hence, and from formulas (2.19), (2.20) for arbitrary step j of the iterative process, it follows:

$$(5.2) \quad \begin{aligned} u_{j1}(x_1, x_2, t) = & \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_2^2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j1}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 - \\ & - \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1 \gamma_2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j2}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 \end{aligned}$$

$$(5.3) \quad \begin{aligned} u_{j2}(x_1, x_2, t) = & -\frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1 \gamma_2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j1}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 + \\ & + \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\gamma_1^2}{(\gamma_1^2 + \gamma_2^2)} \int_0^t e^{-\nu(\gamma_1^2 + \gamma_2^2)(t-\tau)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(\tilde{x}_1 \gamma_1 + \tilde{x}_2 \gamma_2)} f_{j2}(\tilde{x}_1, \tilde{x}_2, \tau) d\tilde{x}_1 d\tilde{x}_2 d\tau \cdot \\ & \cdot e^{-i(x_1 \gamma_1 + x_2 \gamma_2)} d\gamma_1 d\gamma_2 \end{aligned}$$

We convert the Cartesian coordinates to the polar coordinates by formulas:

$x_1 = r \cdot \cos \varphi$; $x_2 = r \cdot \sin \varphi$; $\gamma_1 = \rho \cdot \cos \psi$; $\gamma_2 = \rho \cdot \sin \psi$; $\tilde{x}_1 = \tilde{r} \cdot \cos \tilde{\varphi}$; $\tilde{x}_2 = \tilde{r} \cdot \sin \tilde{\varphi}$;
and obtain from formulas (5.2), (5.3):

$$\begin{aligned}
u_{j1}(r, \varphi, t) = & \frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \sin^2 \psi \int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot \\
& \cdot e^{-i\tilde{r}\rho \cos(\psi-\varphi)} \rho d\rho d\psi - \\
& - \frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \sin \psi \cos \psi \int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot \\
& \cdot e^{-i\tilde{r}\rho \cos(\psi-\varphi)} \rho d\rho d\psi
\end{aligned}
\tag{5.4}$$

$$\begin{aligned}
u_{j2}(r, \varphi, t) = & -\frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \sin \psi \cos \psi \int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot \\
& \cdot e^{-i\tilde{r}\rho \cos(\psi-\varphi)} \rho d\rho d\psi + \\
& + \frac{1}{4\pi^2} \int_0^\infty \int_0^{2\pi} \cos^2 \psi \int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) \tilde{r} d\tilde{r} d\tilde{\varphi} d\tau \cdot \\
& \cdot e^{-i\tilde{r}\rho \cos(\psi-\varphi)} \rho d\rho d\psi
\end{aligned}
\tag{5.5}$$

We have the applied force \vec{f}_j for arbitrary step j of the iterative process:

$$f_{j\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau) = f_{j\tilde{r}}(\tilde{r}, \tau) e^{in_j \tilde{\varphi}} \quad , \quad f_{j\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau) \equiv 0$$

or

$$f_{j\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau) \equiv 0 \quad , \quad f_{j\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau) = f_{j\tilde{\varphi}}(\tilde{r}, \tau) e^{in_j \tilde{\varphi}}$$

where $f_{j\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau)$, $f_{j\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau)$ – radial and tangential components of the applied force.
 n_j - separate circumferential mode, $n_j = 0, 1, 2, 3, \dots$

We take the radial and tangential components of the applied force (5.6), (5.7) with condition (1.18). For the radial component of the applied force we use De Moivre's formulas (A.8) and have:

$$(5.8) \quad \begin{aligned} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) &= f_{j\tilde{r}}(\tilde{r}, \tau) e^{in_j \tilde{\varphi}} \cos \tilde{\varphi} = \frac{1}{2} f_{j\tilde{r}}(\tilde{r}, \tau) (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) \\ f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) &= f_{j\tilde{r}}(\tilde{r}, \tau) e^{in_j \tilde{\varphi}} \sin \tilde{\varphi} = \frac{i}{2} f_{j\tilde{r}}(\tilde{r}, \tau) (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) \end{aligned}$$

We put the applied force components (5.8) in formulas (5.4), (5.5) and find:

$$(5.9) \quad \begin{aligned} u_{jr1}(r, \varphi, t) &= \frac{1}{8\pi^2} \int_0^\infty \int_0^{2\pi} \sin^2 \psi \left[\int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \cdot \right. \\ &\cdot \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} d\tau \left. \right] e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi - \\ &- \frac{i}{8\pi^2} \int_0^\infty \int_0^{2\pi} \sin \psi \cos \psi \left[\int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \cdot \right. \\ &\cdot \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} d\tau \left. \right] e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi \end{aligned}$$

$$(5.10) \quad \begin{aligned} u_{jr2}(r, \varphi, t) &= -\frac{1}{8\pi^2} \int_0^\infty \int_0^{2\pi} \sin \psi \cos \psi \left[\int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \cdot \right. \\ &\cdot \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} d\tau \left. \right] e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi + \\ &+ \frac{i}{8\pi^2} \int_0^\infty \int_0^{2\pi} \cos^2 \psi \left[\int_0^t e^{-\nu \rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \cdot \right. \\ &\cdot \int_0^{2\pi} e^{i\tilde{r}\rho \cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi} \tilde{r} d\tilde{r} d\tau \left. \right] e^{-ir\rho \cos(\psi-\varphi)} \rho d\rho d\psi \end{aligned}$$

Let us denote internal integrals in (5.9), (5.10) as $I_{\pm}(\tilde{r}, \rho, \psi)$:

$$(5.11) \quad I_{\pm}(\tilde{r}, \rho, \psi) = \int_0^{2\pi} e^{i\tilde{r}\rho\cos(\tilde{\varphi}-\psi)} (e^{i(n_j-1)\tilde{\varphi}} \pm e^{i(n_j+1)\tilde{\varphi}}) d\tilde{\varphi}$$

We have two integrals here. Plus (+) is for the first part of each integral (5.9), (5.10) and minus (-) is for the second part.

We substitute $\tilde{\theta}$ for $\tilde{\varphi}$: $\tilde{\theta} = \tilde{\varphi} - \psi$, $d\tilde{\theta} = d\tilde{\varphi}$ and receive:

$$(5.12) \quad I_{\pm}(\tilde{r}, \rho, \psi) = e^{i(n_j-1)\psi} \int_{-\psi}^{2\pi-\psi} e^{i\tilde{r}\rho\cos\tilde{\theta}+i(n_j-1)\tilde{\theta}} d\tilde{\theta} \pm e^{i(n_j+1)\psi} \int_{-\psi}^{2\pi-\psi} e^{i\tilde{r}\rho\cos\tilde{\theta}+i(n_j+1)\tilde{\theta}} d\tilde{\theta}$$

Then we use the Bessel function's integral representation (A.9) and have:

$$(5.13) \quad I_{\pm}(\tilde{r}, \rho, \psi) = 2\pi i^{(n_j-1)} e^{i(n_j-1)\psi} J_{n_j-1}(\tilde{r}\rho) \pm 2\pi i^{(n_j+1)} e^{i(n_j+1)\psi} J_{n_j+1}(\tilde{r}\rho)$$

Let us put $I_{\pm}(\tilde{r}, \rho, \psi)$ from (5.13) in formulas (5.9), (5.10), change order of integration and obtain:

$$(5.14) \quad \begin{aligned} u_{jr1}(r, \varphi, t) &= \frac{1}{8\pi^2} \int_0^\infty \int_0^t e^{-\nu\rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \cdot \\ &\cdot \int_0^{2\pi} \left[\sin^2\psi I_+(\tilde{r}, \rho, \psi) - i \sin\psi \cos\psi I_-(\tilde{r}, \rho, \psi) \right] e^{-i\tilde{r}\rho\cos(\psi-\varphi)} d\psi \tilde{r} d\tilde{r} d\tau \rho d\rho \end{aligned}$$

$$(5.15) \quad \begin{aligned} u_{jr2}(r, \varphi, t) &= \frac{1}{8\pi^2} \int_0^\infty \int_0^t e^{-\nu\rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \cdot \\ &\cdot \int_0^{2\pi} \left[-\sin\psi \cos\psi I_+(\tilde{r}, \rho, \psi) + i \cos^2\psi I_-(\tilde{r}, \rho, \psi) \right] e^{-i\tilde{r}\rho\cos(\psi-\varphi)} d\psi \tilde{r} d\tilde{r} d\tau \rho d\rho \end{aligned}$$

Then we group parts in brackets of formulas (5.14), (5.15), use De Moivre's formulas (A.8) and the Bessel function's properties. And we get:

$$(5.16) \quad u_{jr1}(r, \varphi, t) = -\frac{n_j i^{n_j}}{2\pi} \int_0^\infty \int_0^t e^{-\nu\rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \int_0^{2\pi} \sin\psi e^{-i\tilde{r}\rho\cos(\psi-\varphi)+in_j\psi} d\psi J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho$$

$$(5.17) \quad u_{jr2}(r, \varphi, t) = \frac{n_j i^{n_j}}{2\pi} \int_0^\infty \int_0^t e^{-\nu\rho^2(t-\tau)} \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) \int_0^{2\pi} \cos\psi e^{-i\tilde{r}\rho\cos(\psi-\varphi)+in_j\psi} d\psi J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho$$

We substitute θ for ψ : $\theta = \psi - \varphi$, $d\theta = d\psi$ in the internal integrals of formulas (5.16), (5.17), use De Moivre's formulas (A.8) and the Bessel function's integral representation (A.9) and have from formulas (5.16), (5.17):

$$u_{jr1}(r, \varphi, t) = \frac{n_j}{2} e^{in_j \varphi} \int_0^\infty \int_0^t e^{-\nu \rho^2(t-\tau)} \left[e^{i\varphi} J_{n_j+1}(r\rho) + e^{-i\varphi} J_{n_j-1}(r\rho) \right] \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho \quad (5.18)$$

$$u_{jr2}(r, \varphi, t) = \frac{i n_j}{2} e^{in_j \varphi} \int_0^\infty \int_0^t e^{-\nu \rho^2(t-\tau)} \left[e^{i\varphi} J_{n_j+1}(r\rho) - e^{-i\varphi} J_{n_j-1}(r\rho) \right] \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho \quad (5.19)$$

Let us denote:

$$R_{j,n_j-1,r}(r, t) = \int_0^\infty \int_0^t e^{-\nu \rho^2(t-\tau)} J_{n_j-1}(r\rho) \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho \quad (5.20)$$

$$R_{j,n_j+1,r}(r, t) = \int_0^\infty \int_0^t e^{-\nu \rho^2(t-\tau)} J_{n_j+1}(r\rho) \int_0^\infty f_{j\tilde{r}}(\tilde{r}, \tau) J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho \quad (5.21)$$

Then we have from formulas (5.18), (5.19):

$$u_{jr1}(r, \varphi, t) = \frac{n_j}{2} [R_{j,n_j-1,r}(r, t) e^{i(n_j-1)\varphi} + R_{j,n_j+1,r}(r, t) e^{i(n_j+1)\varphi}] \quad (5.22)$$

$$u_{jr2}(r, \varphi, t) = \frac{i n_j}{2} [R_{j,n_j-1,r}(r, t) e^{i(n_j-1)\varphi} - R_{j,n_j+1,r}(r, t) e^{i(n_j+1)\varphi}] \quad (5.23)$$

Then if $n_j = 0$ it follows from (5.20), (5.21), (5.22), (5.23) that $u_{jr1}(r, \varphi, t) = u_{jr2}(r, \varphi, t) = 0$ and hence $u_1 = u_2 = 0$.

In the equations below we will consider $n_j \geq 1$.

Now we integrate the solution (5.4), (5.5) by the tangential component of the applied force (5.7) (for $n_j \geq 1$). Then we use De Moivre's formulas (A.8) and have:

$$\begin{aligned} f_{j1}(\tilde{r}, \tilde{\varphi}, \tau) &= -f_{j\tilde{\varphi}}(\tilde{r}, \tau) e^{in_j \tilde{\varphi}} \sin \tilde{\varphi} = -\frac{i}{2} f_{j\tilde{\varphi}}(\tilde{r}, \tau) (e^{i(n_j-1)\tilde{\varphi}} - e^{i(n_j+1)\tilde{\varphi}}) \\ f_{j2}(\tilde{r}, \tilde{\varphi}, \tau) &= f_{j\tilde{\varphi}}(\tilde{r}, \tau) e^{in_j \tilde{\varphi}} \cos \tilde{\varphi} = \frac{1}{2} f_{j\tilde{\varphi}}(\tilde{r}, \tau) (e^{i(n_j-1)\tilde{\varphi}} + e^{i(n_j+1)\tilde{\varphi}}) \end{aligned} \quad (5.24)$$

Hence formulas (5.24) are the components f_{j1} and f_{j2} from the tangential applied force (5.7), while formulas (5.8) are the components f_{j1} and f_{j2} from the radial applied force (5.6).

Let us put (5.24) in formulas (5.4), (5.5) and do the operations as we did in (5.9) – (5.23) ($n_j \geq 1$). We consider that $f_{j\tilde{\varphi}}(\tilde{r}, \tau)$ is restricted by condition (1.18) and get:

$$(5.25) \quad R_{j,n_j-1,\varphi}(r,t) = - \int_0^\infty \int_0^t e^{-\nu\rho^2(t-\tau)} J_{n_j-1}(r\rho) \int_0^\infty (f_{j\tilde{\varphi}}(\tilde{r}, \tau) \cdot \tilde{r})'_{\tilde{r}} J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho$$

$$(5.26) \quad R_{j,n_j+1,\varphi}(r,t) = - \int_0^\infty \int_0^t e^{-\nu\rho^2(t-\tau)} J_{n_j+1}(r\rho) \int_0^\infty (f_{j\tilde{\varphi}}(\tilde{r}, \tau) \cdot \tilde{r})'_{\tilde{r}} J_{n_j}(\tilde{r}\rho) d\tilde{r} d\tau d\rho,$$

Here $(\cdot)'_{\tilde{r}} \equiv \frac{\partial}{\partial \tilde{r}}$. Hence we have:

$$(5.27) \quad u_{j\varphi 1}(r, \varphi, t) = - \frac{i}{2} [R_{j,n_j-1,\varphi}(r,t) e^{i(n_j-1)\varphi} + R_{j,n_j+1,\varphi}(r,t) e^{i(n_j+1)\varphi}]$$

$$(5.28) \quad u_{j\varphi 2}(r, \varphi, t) = \frac{1}{2} [R_{j,n_j-1,\varphi}(r,t) e^{i(n_j-1)\varphi} - R_{j,n_j+1,\varphi}(r,t) e^{i(n_j+1)\varphi}]$$

We have obtain formulas (5.8) - (5.28) for an arbitrary step j of the iterative process and the applied forces (5.6) or (5.7).

Now we investigate the first step ($j = 1$, $n_1 = n = 1, 2, 3, \dots$) of the iterative process with the particular radial applied force $f_1(x, t)$ [look at (2.24)]:

$$f_{1\tilde{r}}(\tilde{r}, \tilde{\varphi}, \tau) = f_{1\tilde{r}}(\tilde{r}, \tau) e^{in\tilde{\varphi}}, \quad f_{1\tilde{\varphi}}(\tilde{r}, \tilde{\varphi}, \tau) \equiv 0$$

$$(5.29) \quad f_{1\tilde{r}}(\tilde{r}, \tau) = F_n \tilde{r}^{n+1} e^{-\mu_n^2 \tilde{r}^2} f_{1\tau}(\tau)$$

F_n, μ_n - constants. , $0 < F_n < \infty$, $1 < \mu_n < \infty$.

Let us put the particular radial applied force (5.29) in formulas (5.20), (5.21) and for the internal integral we have by using formula (A.11), [11]:

$$(5.30) \quad I(\rho, \tau) = \int_0^\infty f_{1\tilde{r}}(\tilde{r}, \tau) J_n(\tilde{r}\rho) d\tilde{r} = F_n f_{1\tau}(\tau) \int_0^\infty \tilde{r}^{n+1} e^{-\mu_n^2 \tilde{r}^2} J_n(\tilde{r}\rho) d\tilde{r} = \frac{F_n f_{1\tau}(\tau) \rho^n}{(2\mu_n^2)^{n+1}} e^{-\frac{\rho^2}{4\mu_n^2}}$$

Now we put $I(\rho, \tau)$ from formula (5.30) in formulas (5.20), (5.21), change the order of integration and have by using formula (A.12), [11]:

$$(5.31) \quad \begin{aligned} R_{1,n-1,r}(r,t) &= F_n \int_0^t f_{1\tau}(\tau) \int_0^\infty e^{-[\nu(t-\tau) + \frac{1}{4\mu_n^2}]\rho^2} \frac{\rho^n}{(2\mu_n^2)^{n+1}} J_{n-1}(r\rho) d\rho d\tau = \\ &= \frac{F_n r^{n-1}}{2\mu_n^2} \int_0^t \frac{f_{1\tau}(\tau) \Phi(n+2, n+2; \frac{-\mu_n^2 r^2}{[4\mu_n^2 \nu(t-\tau) + 1]})}{[4\mu_n^2 \nu(t-\tau) + 1]^n} d\tau \end{aligned}$$

$$\begin{aligned}
R_{1,n+1,r}(r,t) &= F_n \int_0^t f_{1\tau}(\tau) \int_0^\infty e^{-[\nu(t-\tau)+\frac{1}{4\mu_n^2}]\rho^2} \frac{\rho^n}{(2\mu_n^2)^{n+1}} J_{n+1}(r\rho) d\rho d\tau = \\
(5.32) \quad &= \frac{F_n r^{n+1}}{2(n+1)} \int_0^t \frac{f_{1\tau}(\tau) \Phi(n+1, n+2; \frac{-\mu_n^2 r^2}{[4\mu_n^2 \nu(t-\tau)+1]})}{[4\mu_n^2 \nu(t-\tau)+1]^{n+1}} d\tau
\end{aligned}$$

Here $\Phi(a, c; x)$ is a confluent hypergeometric function [10].

We substitute y for τ : $y = \frac{1}{[4\mu_n^2 \nu(t-\tau)+1]}$, $dy = \frac{4\mu_n^2 \nu}{[4\mu_n^2 \nu(t-\tau)+1]^2} d\tau$ and receive:

$$(5.33) \quad R_{1,n-1,r}(r,t) = \frac{F_n r^{n-1}}{8\mu_n^4 \nu} \int_{\frac{1}{[4\mu_n^2 \nu t+1]}}^1 f_{1\tau}(y) \cdot y^{n-2} \cdot \Phi(n+2, n+2; -\mu_n^2 r^2 y) dy$$

$$(5.34) \quad R_{1,n+1,r}(r,t) = \frac{F_n r^{n+1}}{8\mu_n^2 \nu(n+1)} \int_{\frac{1}{[4\mu_n^2 \nu t+1]}}^1 f_{1\tau}(y) \cdot y^{n-1} \cdot \Phi(n+1, n+2; -\mu_n^2 r^2 y) dy$$

Let us denote $f_{1\tau}(y) = y^2$ and get:

$$(5.35) \quad R_{1,n-1,r}(r,t) = \frac{F_n r^{n-1}}{8\mu_n^4 \nu} \int_{\frac{1}{[4\mu_n^2 \nu t+1]}}^1 y^n \cdot \Phi(n+2, n+2; -\mu_n^2 r^2 y) dy$$

$$(5.36) \quad R_{1,n+1,r}(r,t) = \frac{F_n r^{n+1}}{8\mu_n^2 \nu(n+1)} \int_{\frac{1}{[4\mu_n^2 \nu t+1]}}^1 y^{n+1} \cdot \Phi(n+1, n+2; -\mu_n^2 r^2 y) dy$$

We use formula (A.13) for integrand in the integral (5.35) and formula (A.14) for integrand in the integral (5.36) [10], integrate and then we have:

$$\begin{aligned}
R_{1,n-1,r}(r,t) &= \frac{F_n r^{n-1}}{8\mu_n^4 \nu(n+1)} \left[\Phi(n+1, n+2; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t+1)})}{(4\mu_n^2 \nu t+1)^{n+1}} \right] \\
(5.37) \quad &
\end{aligned}$$

$$\begin{aligned}
R_{1,n+1,r}(r,t) &= \frac{F_n r^{n+1}}{8\mu_n^2 \nu(n+1)(n+2)} \left[\Phi(n+1, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t+1)})}{(4\mu_n^2 \nu t+1)^{n+2}} \right] \\
(5.38) \quad &
\end{aligned}$$

Hence, and from formulas (5.22), (5.23) it follows for $j = 1$:

$$(5.39) \quad u_{1r1}(r, \varphi, t) = \frac{n}{2} [R_{1,n-1,r}(r,t) e^{i(n-1)\varphi} + R_{1,n+1,r}(r,t) e^{i(n+1)\varphi}]$$

$$(5.40) \quad u_{1r2}(r, \varphi, t) = \frac{i n}{2} [R_{1,n-1,r}(r, t) e^{i(n-1)\varphi} - R_{1,n+1,r}(r, t) e^{i(n+1)\varphi}]$$

We continue operations and get:

$$(5.41) \quad u_{1r}(r, \varphi, t) = \frac{n}{2} [R_{1,n-1,r}(r, t) + R_{1,n+1,r}(r, t)] e^{in\varphi}$$

$$(5.42) \quad u_{1\varphi}(r, \varphi, t) = \frac{i n}{2} [R_{1,n-1,r}(r, t) - R_{1,n+1,r}(r, t)] e^{in\varphi}$$

$u_{1r}(r, \varphi, t)$, $u_{1\varphi}(r, \varphi, t)$ are the radial and tangential components of the velocity \vec{u}_1 .

We use the properties of the confluent hypergeometric function $\Phi(a, c; x)$ and have from formulas (5.37)–(5.42):

$$(5.43) \quad \begin{aligned} \lim_{t \rightarrow 0} u_{1r1}(r, \varphi, t) &= 0; & \lim_{t \rightarrow 0} u_{1r2}(r, \varphi, t) &= 0; \\ \lim_{t \rightarrow 0} u_{1r}(r, \varphi, t) &= 0; & \lim_{t \rightarrow 0} u_{1\varphi}(r, \varphi, t) &= 0; \end{aligned}$$

Hence we have velocity \vec{u}_1 satisfies the initial conditions (5.1).

Then we use the asymptotic properties of the confluent hypergeometric function $\Phi(a, c; x)$ [10] and from formulas (5.37) – (5.42) we have velocity \vec{u}_1 satisfies conditions (1.16) (for $r \rightarrow \infty$).

Let us continue investigation for the second step ($j = 2$) of the iterative process.

Find $\vec{f}_2^*(r, \varphi, t) = \{f_{21}^*, f_{22}^*\}$ - the first correction of the particular radial applied force $f_1(x, t)$ (5.29).

We have for \vec{f}_2^* from formula (2.26):

$$(5.44) \quad f_{21}^* = u_{1r1} \frac{\partial u_{1r1}}{\partial x_1} + u_{1r2} \frac{\partial u_{1r1}}{\partial x_2}$$

$$(5.45) \quad f_{22}^* = u_{1r1} \frac{\partial u_{1r2}}{\partial x_1} + u_{1r2} \frac{\partial u_{1r2}}{\partial x_2}$$

where u_{1r1} , u_{1r2} are the components of \vec{u}_1 and were taken from formulas (5.39), (5.40).

We have here:

$$\begin{aligned} \frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_1} &= \frac{\partial u_{1r1}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_1} + \frac{\partial u_{1r1}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_1} \\ \frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_2} &= \frac{\partial u_{1r1}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_2} + \frac{\partial u_{1r1}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_2} \\ \frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_1} &= \frac{\partial u_{1r2}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_1} + \frac{\partial u_{1r2}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_1} \end{aligned}$$

$$\begin{aligned}
\frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_2} &= \frac{\partial u_{1r2}(r, \varphi, t)}{\partial r} \frac{\partial r}{\partial x_2} + \frac{\partial u_{1r2}(r, \varphi, t)}{\partial \varphi} \frac{\partial \varphi}{\partial x_2} \\
\frac{\partial r}{\partial x_1} &= \cos \varphi, \quad \frac{\partial \varphi}{\partial x_1} = -\frac{\sin \varphi}{r}, \quad \frac{\partial r}{\partial x_2} = \sin \varphi, \quad \frac{\partial \varphi}{\partial x_2} = \frac{\cos \varphi}{r}
\end{aligned}
\tag{5.46}$$

Then we use formulas (5.39), (5.40) for $u_{1r1}(r, \varphi, t)$, $u_{1r2}(r, \varphi, t)$ and have from (5.46):

$$\begin{aligned}
\frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_1} &= \frac{n}{2} \left\{ [R'_{1,n-1,r}(r, t) e^{i(n-1)\varphi} + R'_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \cos \varphi + \right. \\
&\quad \left. + i [(n-1)R_{1,n-1,r}(r, t) e^{i(n-1)\varphi} + (n+1)R_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \left(-\frac{\sin \varphi}{r}\right) \right\} \\
\frac{\partial u_{1r1}(r, \varphi, t)}{\partial x_2} &= \frac{n}{2} \left\{ [R'_{1,n-1,r}(r, t) e^{i(n-1)\varphi} + R'_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \sin \varphi + \right. \\
&\quad \left. + i [(n-1)R_{1,n-1,r}(r, t) e^{i(n-1)\varphi} + (n+1)R_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \frac{\cos \varphi}{r} \right\} \\
\frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_1} &= \frac{in}{2} \left\{ [R'_{1,n-1,r}(r, t) e^{i(n-1)\varphi} - R'_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \cos \varphi + \right. \\
&\quad \left. + i [(n-1)R_{1,n-1,r}(r, t) e^{i(n-1)\varphi} - (n+1)R_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \left(-\frac{\sin \varphi}{r}\right) \right\} \\
\frac{\partial u_{1r2}(r, \varphi, t)}{\partial x_2} &= \frac{in}{2} \left\{ [R'_{1,n-1,r}(r, t) e^{i(n-1)\varphi} - R'_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \sin \varphi + \right. \\
&\quad \left. + i [(n-1)R_{1,n-1,r}(r, t) e^{i(n-1)\varphi} - (n+1)R_{1,n+1,r}(r, t) e^{i(n+1)\varphi}] \frac{\cos \varphi}{r} \right\}
\end{aligned}
\tag{5.47}$$

where

$$\begin{aligned}
R'_{1,n-1,r}(r, t) &= \frac{\partial R_{1,n-1,r}(r, t)}{\partial r} = \frac{F_n(n-1)r^{n-2}}{8\mu_n^4\nu(n+1)} \left[\Phi(n+1, n+2; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t+1)})}{(4\mu_n^2\nu t+1)^{n+1}} \right] - \\
&\quad - \frac{F_n r^n}{4\mu_n^2\nu(n+2)} \left[\Phi(n+2, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+2, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t+1)})}{(4\mu_n^2\nu t+1)^{n+2}} \right]
\end{aligned}
\tag{5.48}$$

$$R'_{1,n+1,r}(r, t) = \frac{\partial R_{1,n+1,r}(r, t)}{\partial r} = \frac{F_n r^n}{8\mu_n^2\nu(n+2)} \left[\Phi(n+1, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t+1)})}{(4\mu_n^2\nu t+1)^{n+2}} \right] -$$

$$(5.49) \quad -\frac{F_n r^{n+2}}{4\nu(n+2)(n+3)} \left[\Phi(n+2, n+4; -\mu_n^2 r^2) - \frac{\Phi(n+2, n+4; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+3}} \right]$$

Let us put u_{1r1} , u_{1r2} , $\frac{\partial u_{1r1}}{\partial x_1}$, $\frac{\partial u_{1r1}}{\partial x_2}$, $\frac{\partial u_{1r2}}{\partial x_1}$, $\frac{\partial u_{1r2}}{\partial x_2}$ from formulas (5.39), (5.40), (5.47) in formulas (5.44), (5.45) for f_{21}^* , f_{22}^* .

After compliting appropriate operations we have:

$$(5.50) \quad f_{21}^*(r, \varphi, t) = \frac{n^2}{2^2} [T_{2,2n-1,r}(r, t) e^{i(2n-1)\varphi} + T_{2,2n+1,r}(r, t) e^{i(2n+1)\varphi}]$$

$$(5.51) \quad f_{22}^*(r, \varphi, t) = \frac{in^2}{2^2} [T_{2,2n-1,r}(r, t) e^{i(2n-1)\varphi} - T_{2,2n+1,r}(r, t) e^{i(2n+1)\varphi}]$$

where

$$(5.52) \quad \begin{aligned} T_{2,2n-1,r}(r, t) &= [R_{1,n-1,r}(r, t) + R_{1,n+1,r}(r, t)] R'_{1,n-1,r}(r, t) - \\ &\quad - \frac{(n-1)}{r} R_{1,n-1,r}(r, t) [R_{1,n-1,r}(r, t) - R_{1,n+1,r}(r, t)] \\ T_{2,2n+1,r}(r, t) &= [R_{1,n-1,r}(r, t) + R_{1,n+1,r}(r, t)] R'_{1,n+1,r}(r, t) - \\ &\quad - \frac{(n+1)}{r} R_{1,n+1,r}(r, t) [R_{1,n-1,r}(r, t) - R_{1,n+1,r}(r, t)] \end{aligned}$$

For radial f_{2r}^* and tangential $f_{2\varphi}^*$ components of the first correction $\vec{f}_2^*(r, \varphi, t)$ of the particular radial applied force we have:

$$(5.53) \quad f_{2r}^*(r, \varphi, t) = \frac{n^2}{2^2} [T_{2,2n-1,r}(r, t) + T_{2,2n+1,r}(r, t)] e^{i2n\varphi} = \frac{n^2}{2^2} T_{2,2n,r}(r, t) e^{i2n\varphi}$$

$$(5.54) \quad f_{2\varphi}^*(r, \varphi, t) = \frac{in^2}{2^2} [T_{2,2n-1,r}(r, t) - T_{2,2n+1,r}(r, t)] e^{i2n\varphi} = \frac{in^2}{2^2} T_{2,2n,\varphi}(r, t) e^{i2n\varphi}$$

where (see (5.52))

$$T_{2,2n,r}(r, t) = [R_{1,n-1,r}(r, t) + R_{1,n+1,r}(r, t)] [R'_{1,n-1,r}(r, t) + R'_{1,n+1,r}(r, t)] -$$

$$\begin{aligned}
& -\frac{1}{r}[(n-1)R_{1,n-1,r}(r,t) + (n+1)R_{1,n+1,r}(r,t)][R_{1,n-1,r}(r,t) - R_{1,n+1,r}(r,t)] \\
& T_{2,2n,\varphi}(r,t) = [R_{1,n-1,r}(r,t) + R_{1,n+1,r}(r,t)][R'_{1,n-1,r}(r,t) - R'_{1,n+1,r}(r,t)] - \\
& -\frac{1}{r}[(n-1)R_{1,n-1,r}(r,t) - (n+1)R_{1,n+1,r}(r,t)][R_{1,n-1,r}(r,t) - R_{1,n+1,r}(r,t)]
\end{aligned}
\tag{5.55}$$

We use formulas (5.37), (5.38) for $R_{1,n-1,r}(r,t)$, $R_{1,n+1,r}(r,t)$ and (5.48), (5.49) for $R'_{1,n-1,r}(r,t)$, $R'_{1,n+1,r}(r,t)$ then do the appropriate operations for $T_{2,2n-1,r}(r,t)$, $T_{2,2n+1,r}(r,t)$, using formula (A.15), and get:

$$\begin{aligned}
T_{2,2n,r}(r,t) &= \frac{-F_n^2 \cdot r^{2n-1}}{16\mu_n^6 \nu^2 (n+1)^2 (n+2)} \left[\Phi(n+1, n+2; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+1}} \right] \\
&\cdot \left[\Phi(n+1, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+2}} \right]
\end{aligned}
\tag{5.56}$$

$$\begin{aligned}
T_{2,2n,\varphi}(r,t) &= \frac{-F_n^2 \cdot r^{2n-1}}{16\mu_n^6 \nu^2 (n+1)(n+2)} \left[\Phi(n, n+2; -\mu_n^2 r^2) - \frac{\Phi(n, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+1}} \right] \\
&\cdot \left[\Phi(n+2, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+2, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+2}} \right] + \\
&+ \frac{F_n^2 \cdot n \cdot r^{2n-1}}{16\mu_n^6 \nu^2 (n+1)^2 (n+2)} \left[\Phi(n+1, n+2; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+1}} \right] \\
&\cdot \left[\Phi(n+1, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+2}} \right]
\end{aligned}
\tag{5.57}$$

By comparing particular radial applied force \vec{f}_1 from (5.29) with the first correction \vec{f}_2^* from ((5.53) – (5.57)) of this particular radial applied force we have:

$$|\vec{f}_2^*| < < |\vec{f}_1|
\tag{5.58}$$

with condition

$$F_n \leq \frac{1}{n}
\tag{5.59}$$

After the first step of the iterative process ($j = 1$) we had the velocity \vec{u}_1 - formulas (5.41) , (5.42). Now we will calculate \vec{u}_2^* - the first correction of the velocity \vec{u}_1 . Solution of this problem has two stages. On the first stage we find the part of the first correction u_{2r}^* , corresponding to the first correction f_{2r}^* from formula (5.53) of the applied force:

$$(5.60) \quad f_{2r}^*(r, \varphi, t) = \frac{n^2}{2^2} T_{2,2n,r}(r, t) e^{i 2n\varphi} \quad , \quad f_{2\varphi}^*(r, \varphi, t) \equiv 0$$

On the second stage we calculate the other part of the first correction $u_{2\varphi}^*$, corresponding to the first correction $f_{2\varphi}^*$ from formula (5.54) of the applied force:

$$(5.61) \quad f_{2r}^*(r, \varphi, t) \equiv 0 \quad , \quad f_{2\varphi}^*(r, \varphi, t) = \frac{in^2}{2^2} T_{2,2n,\varphi}(r, t) e^{i 2n\varphi}$$

In other words

$$(5.62) \quad \vec{u}_2^* = u_{2r}^* + u_{2\varphi}^*, \quad u_{2r}^* = \{u_{2r1}^*, u_{2r2}^*\}, \quad u_{2\varphi}^* = \{u_{2\varphi1}^*, u_{2\varphi2}^*\}.$$

First stage: After completing appropriate operations we have from formula (5.56):

$$(5.63) \quad \begin{aligned} T_{2,2n,r}(r, t) = & \frac{-F_n^2 \cdot r^{2n-1}}{16\mu_n^6 \nu^2 (n+1)^2 (n+2)} \left[\Phi(n+1, n+2; -\mu_n^2 r^2) \cdot \Phi(n+1, n+3; -\mu_n^2 r^2) - \right. \\ & - \frac{1}{(4\mu_n^2 \nu t + 1)^{n+2}} \cdot \Phi(n+1, n+2; -\mu_n^2 r^2) \cdot \Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)}) - \\ & - \frac{1}{(4\mu_n^2 \nu t + 1)^{n+1}} \cdot \Phi(n+1, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)}) \cdot \Phi(n+1, n+3; -\mu_n^2 r^2) + \\ & \left. + \frac{1}{(4\mu_n^2 \nu t + 1)^{2n+3}} \cdot \Phi(n+1, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)}) \cdot \Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)}) \right] \end{aligned}$$

We take formulas (5.22) , (5.23) for components $u_{jr1}(r, \varphi, t)$, $u_{jr2}(r, \varphi, t)$ and formulas (5.20), (5.21) for $R_{j,n_j-1,r}(r, t)$, $R_{j,n_j+1,r}(r, t)$ for $j = 2$ and then formulas (5.60), (5.63) for $f_{2r}^*(r, \varphi, t)$, $T_{2,2n,r}(r, t)$. We do appropriate operations and have:

$$(5.64) \quad u_{2r1}^*(r, \varphi, t) = n [R_{2,2n-1,r}(r, t) e^{i(2n-1)\varphi} + R_{2,2n+1,r}(r, t) e^{i(2n+1)\varphi}]$$

$$(5.65) \quad u_{2r2}^*(r, \varphi, t) = i n [R_{2,2n-1,r}(r, t) e^{i(2n-1)\varphi} - R_{2,2n+1,r}(r, t) e^{i(2n+1)\varphi}]$$

After changing the order of integration we receive:

$$(5.66) \quad R_{2,2n-1,r}(r, t) = \frac{n^2}{2^2} \int_0^t \int_0^\infty T_{2,2n,r}(\tilde{r}, \tau) \int_0^\infty e^{-\nu \rho^2(t-\tau)} J_{2n-1}(r\rho) J_{2n}(\tilde{r}\rho) \rho d\rho \tilde{r} d\tau$$

$$(5.67) \quad R_{2,2n+1,r}(r, t) = \frac{n^2}{2^2} \int_0^t \int_0^\infty T_{2,2n,r}(\tilde{r}, \tau) \int_0^\infty e^{-\nu \rho^2(t-\tau)} J_{2n+1}(r\rho) J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau$$

Second stage: After operations with $T_{2,2n,\varphi}(r, t)$ from formula (5.57) and using formula (A.15), we obtain:

$$(5.68) \quad (T_{2,2n,\varphi}(r, t) \cdot r)'_r = \frac{\partial(T_{2,2n,\varphi}(r, t) \cdot r)}{\partial r} = T_{2,2n,\varphi,\varphi}(r, t) + T_{2,2n,\varphi,r}(r, t)$$

We denote here

$$(5.69) \quad \begin{aligned} T_{2,2n,\varphi,\varphi}(r, t) = & \frac{-F_n^2 \cdot n \cdot r^{2n+1}}{8\mu_n^4 \nu^2 (n+1)(n+2)^2} \left[\Phi(n+1, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+2}} \right] \cdot \\ & \cdot \left[\Phi(n+2, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+2, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+2}} \right] + \\ & + \frac{F_n^2 \cdot r^{2n+1}}{8\mu_n^4 \nu^2 (n+1)(n+3)} \left[\Phi(n, n+2; -\mu_n^2 r^2) - \frac{\Phi(n, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+1}} \right] \cdot \\ & \cdot \left[\Phi(n+3, n+4; -\mu_n^2 r^2) - \frac{\Phi(n+3, n+4; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+3}} \right] \end{aligned}$$

and

$$(5.70) \quad \begin{aligned} T_{2,2n,\varphi,r}(r, t) = & \frac{-F_n^2 \cdot n \cdot r^{2n-1}}{8\mu_n^6 \nu^2 (n+1)^2 (n+2)} \left[\Phi(n+1, n+2; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+1}} \right] \cdot \\ & \cdot \left[\Phi(n+1, n+3; -\mu_n^2 r^2) - \frac{\Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2 \nu t + 1)})}{(4\mu_n^2 \nu t + 1)^{n+2}} \right] = 2n \cdot T_{2,2n,r}(r, t) \end{aligned}$$

$T_{2,2n,r}(r, t)$ is taken from formulas (5.56) and (5.63).

Then we do several operations and have from formula (5.69):

$$T_{2,2n,\varphi,\varphi}(r, t) = \frac{-F_n^2 \cdot n \cdot r^{2n+1}}{8\mu_n^4 \nu^2 (n+1)(n+2)^2} \left[\Phi(n+1, n+3; -\mu_n^2 r^2) \cdot \Phi(n+2, n+3; -\mu_n^2 r^2) - \right.$$

$$\begin{aligned}
& -\frac{1}{(4\mu_n^2\nu t + 1)^{n+2}} \cdot \Phi(n+1, n+3; -\mu_n^2 r^2) \cdot \Phi(n+2, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) - \\
& -\frac{1}{(4\mu_n^2\nu t + 1)^{n+2}} \cdot \Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) \cdot \Phi(n+2, n+3; -\mu_n^2 r^2) + \\
& +\frac{1}{(4\mu_n^2\nu t + 1)^{2n+4}} \cdot \Phi(n+1, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) \cdot \Phi(n+2, n+3; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) \Big] + \\
& +\frac{F_n^2 \cdot r^{2n+1}}{8\mu_n^4\nu^2(n+1)(n+3)} \Big[\Phi(n, n+2; -\mu_n^2 r^2) \cdot \Phi(n+3, n+4; -\mu_n^2 r^2) - \\
& -\frac{1}{(4\mu_n^2\nu t + 1)^{n+3}} \cdot \Phi(n, n+2; -\mu_n^2 r^2) \cdot \Phi(n+3, n+4; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) - \\
& -\frac{1}{(4\mu_n^2\nu t + 1)^{n+1}} \cdot \Phi(n, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) \cdot \Phi(n+3, n+4; -\mu_n^2 r^2) + \\
& +\frac{1}{(4\mu_n^2\nu t + 1)^{2n+4}} \cdot \Phi(n, n+2; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) \cdot \Phi(n+3, n+4; -\frac{\mu_n^2 r^2}{(4\mu_n^2\nu t + 1)}) \Big]
\end{aligned}
\tag{5.71}$$

We transform formulas (5.27), (5.28) for components $u_{j\varphi 1}(r, \varphi)$, $u_{j\varphi 2}(r, \varphi)$ and formulas (5.25), (5.26) for $R_{j, n_j-1, \varphi}(r)$, $R_{j, n_j+1, \varphi}(r)$ for $j = 2$ and then use formula (5.61) for $f_{2\varphi}^*(r, \varphi, t)$. We do several operations and have:

$$u_{2\varphi 1}^*(r, \varphi, t) = -\frac{i}{2} [R_{2, 2n-1, \varphi}(r, t) e^{i(2n-1)\varphi} + R_{2, 2n+1, \varphi}(r, t) e^{i(2n+1)\varphi}]
\tag{5.72}$$

$$u_{2\varphi 2}^*(r, \varphi, t) = \frac{1}{2} [R_{2, 2n-1, \varphi}(r, t) e^{i(2n-1)\varphi} - R_{2, 2n+1, \varphi}(r, t) e^{i(2n+1)\varphi}]
\tag{5.73}$$

After changing the order of integration we obtain:

$$R_{2, 2n-1, \varphi}(r, t) = -\frac{in^2}{2^2} \int_0^t \int_0^\infty (T_{2, 2n, \varphi}(\tilde{r}, \tau) \cdot \tilde{r})'_{\tilde{r}} \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n-1}(r\rho) J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau
\tag{5.74}$$

$$R_{2, 2n+1, \varphi}(r, t) = -\frac{in^2}{2^2} \int_0^t \int_0^\infty (T_{2, 2n, \varphi}(\tilde{r}, \tau) \cdot \tilde{r})'_{\tilde{r}} \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n+1}(r\rho) J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau
\tag{5.75}$$

Then we take $(T_{2, 2n, \varphi}(\tilde{r}, \tau) \cdot \tilde{r})'_{\tilde{r}}$ from formula (5.68) and with use of formula (5.70) put it in formulas (5.74), (5.75) and have:

$$R_{2, 2n-1, \varphi}(r, t) = -\frac{in^2}{2^2} \int_0^t \int_0^\infty T_{2, 2n, \varphi}(\tilde{r}, \tau) \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n-1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau -$$

$$-\frac{in^3}{2} \int_0^t \int_0^\infty T_{2,2n,r}(\tilde{r}, \tau) \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n-1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau = R_{2,2n-1,\varphi,\varphi}(r, t) - 2niR_{2,2n-1,r}(r, t) \quad (5.76)$$

$$R_{2,2n+1,\varphi}(r, t) = -\frac{in^2}{2^2} \int_0^t \int_0^\infty T_{2,2n,\varphi,\varphi}(\tilde{r}, \tau) \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n+1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau - \\ -\frac{in^3}{2} \int_0^t \int_0^\infty T_{2,2n,r}(\tilde{r}, \tau) \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n+1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau = R_{2,2n+1,\varphi,\varphi}(r, t) - 2niR_{2,2n+1,r}(r, t) \quad (5.77)$$

where

$$(5.78) \quad R_{2,2n-1,\varphi,\varphi}(r, t) = -\frac{in^2}{2^2} \int_0^t \int_0^\infty T_{2,2n,\varphi,\varphi}(\tilde{r}, \tau) \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n-1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau$$

$$(5.79) \quad R_{2,2n+1,\varphi,\varphi}(r, t) = -\frac{in^2}{2^2} \int_0^t \int_0^\infty T_{2,2n,\varphi,\varphi}(\tilde{r}, \tau) \int_0^\infty e^{-\nu\rho^2(t-\tau)} J_{2n+1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho d\tilde{r} d\tau$$

and $R_{2,2n-1,r}(r, t)$, $R_{2,2n+1,r}(r, t)$ we take from formulas (5.66), (5.67).

Then we use formulas (5.62), (5.64), (5.65), (5.72), (5.73) and have:

$$u_{21}^*(r, \varphi, t) = u_{2r1}^*(r, \varphi, t) + u_{2\varphi1}^*(r, \varphi, t) = \\ = [nR_{2,2n-1,r}(r, t) - \frac{i}{2}R_{2,2n-1,\varphi}(r, t)] e^{i(2n-1)\varphi} + [nR_{2,2n+1,r}(r, t) - \frac{i}{2}R_{2,2n+1,\varphi}(r, t)] e^{i(2n+1)\varphi} \quad (5.80)$$

$$u_{22}^*(r, \varphi, t) = u_{2r2}^*(r, \varphi, t) + u_{2\varphi2}^*(r, \varphi, t) = \\ = i [nR_{2,2n-1,r}(r, t) - \frac{i}{2}R_{2,2n-1,\varphi}(r, t)] e^{i(2n-1)\varphi} - i [nR_{2,2n+1,r}(r, t) - \frac{i}{2}R_{2,2n+1,\varphi}(r, t)] e^{i(2n+1)\varphi} \quad (5.81)$$

We obtain by performing appropriate transformations:

$$u_{2r}^*(r, \varphi, t) = \left\{ [nR_{2,2n-1,r}(r, t) - \frac{i}{2}R_{2,2n-1,\varphi}(r, t)] + [nR_{2,2n+1,r}(r, t) - \frac{i}{2}R_{2,2n+1,\varphi}(r, t)] \right\} e^{i2n\varphi} \quad (5.82)$$

$$u_{2\varphi}^*(r, \varphi, t) = i \left\{ \left[nR_{2,2n-1,r}(r, t) - \frac{i}{2}R_{2,2n-1,\varphi}(r, t) \right] - \left[nR_{2,2n+1,r}(r, t) - \frac{i}{2}R_{2,2n+1,\varphi}(r, t) \right] \right\} e^{i2n\varphi} \quad (5.83)$$

Here $u_{2r}^*(r, \varphi, t)$, $u_{2\varphi}^*(r, \varphi, t)$ are the radial and tangential components of the first correction \vec{u}_2^* of the velocity \vec{u}_1 . $R_{2,2n-1,r}(r, t)$, $R_{2,2n+1,r}(r, t)$ are taken from formulas (5.66), (5.67).

$$\begin{aligned} R_{2,2n-1,\varphi}(r, t) &= R_{2,2n-1,\varphi,\varphi}(r, t) - 2niR_{2,2n-1,r}(r, t) \\ R_{2,2n+1,\varphi}(r, t) &= R_{2,2n+1,\varphi,\varphi}(r, t) - 2niR_{2,2n+1,r}(r, t) \end{aligned} \quad (5.84)$$

and $R_{2,2n-1,\varphi,\varphi}(r, t)$, $R_{2,2n+1,\varphi,\varphi}(r, t)$ are taken from formulas (5.78), (5.79).

Then we do appropriate operations and have from formulas (5.82), (5.83):

$$u_{2r}^*(r, \varphi, t) = -\frac{i}{2} [R_{2,2n-1,\varphi,\varphi}(r, t) + R_{2,2n+1,\varphi,\varphi}(r, t)] e^{i2n\varphi} \quad (5.85)$$

$$u_{2\varphi}^*(r, \varphi, t) = \frac{1}{2} [R_{2,2n-1,\varphi,\varphi}(r, t) - R_{2,2n+1,\varphi,\varphi}(r, t)] e^{i2n\varphi} \quad (5.86)$$

From formulas (5.80), (5.81) with properties of $R_{2,2n-1,r}(r, t)$, $R_{2,2n+1,r}(r, t)$, $R_{2,2n-1,\varphi}(r, t)$, $R_{2,2n+1,\varphi}(r, t)$ it follows:

$$\lim_{t \rightarrow 0} u_{21}^*(r, \varphi, t) = 0; \quad \lim_{t \rightarrow 0} u_{22}^*(r, \varphi, t) = 0; \quad (5.87)$$

$$\lim_{t \rightarrow 0} u_{2r}^*(r, \varphi, t) = 0; \quad \lim_{t \rightarrow 0} u_{2\varphi}^*(r, \varphi, t) = 0; \quad (5.88)$$

In other words the velocity $\vec{u}_2 = \vec{u}_1 - \vec{u}_2^*$ [look at (2.28)] satisfies the initial conditions (5.1). We use the asymptotic properties of the confluent hypergeometric function $\Phi(a, c; x)$ and have from formulas (5.66), (5.67), (5.78), (5.79): the first correction \vec{u}_2^* and therefore the velocity \vec{u}_2 satisfies conditions (1.16) (for $r \rightarrow \infty$).

By comparing the solution \vec{u}_1 from (5.39), (5.40) or (5.41), (5.42) of the first step of iterative process with the first correction \vec{u}_2^* from (5.80), (5.81) or (5.85), (5.86), which is received on the second step of iterative process, we see that

$$|\vec{u}_2^*| \ll |\vec{u}_1| \quad (5.89)$$

with conditions

$$(5.90) \quad F_n \leq \frac{1}{n}$$

By continuing this iterative process we can obtain next parts $\vec{u}_3^*, \vec{u}_4^*, \dots$, of the converging series for \vec{u} . For arbitrary step j of the iterative process we have by using formula (2.43):

$$(5.91) \quad \vec{u}_j = \vec{u}_1 - \sum_{l=2}^j \vec{u}_l^*$$

and then:

$$(5.92) \quad \lim_{j \rightarrow \infty} \vec{u}_j = \vec{u}$$

where \vec{u} is the solution of the problem (1.1) – (1.6).

Below we provide numerical analysis of these results for the following values of problem's parameters:

Circumferential modes $n = 1, 2, 3, 4, 5$.

$\mu_n = 1$ ($n = 1, 2, 3, 4, 5$).

Results were obtained for functions

$\vec{u}_1 - (5.39), (5.40)$ or $(5.41), (5.42)$ with calculations of the confluent hypergeometric functions [10];

$\vec{u}_2^* - (5.80), (5.81)$ or $(5.85), (5.86)$ by using numerical integration of the triple integrals (5.78), (5.79).

Each of those integrals is computed as an iterated integral.

Let us consider first the calculation of the inner integrals from (5.78), (5.79):

$$(5.93) \quad I_{\pm}(r, \tilde{r}, t, \tau) = \int_0^\infty e^{-\nu \rho^2(t-\tau)} J_{2n_{\pm}+1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho$$

For condition $t < \tau$ the integrand is diminishing fast enough. It is easy to find upper limit of integration, so we can substitute integral (5.93) for

$$(5.94) \quad I_{\pm}(r, \tilde{r}, t, \tau) = \int_0^{A_1} e^{-\nu \rho^2(t-\tau)} J_{2n_{\pm}+1}(r\rho) \cdot J_{2n}(\tilde{r}\rho) d\rho,$$

where $(0 < A_1 = 200 < \infty)$, and hence we are integrating over the finite interval. For additional check let us increase A_1 in 1.5 times and change the number of steps of integration n_1 (from 4001 to 6001). We see that the difference in result values of integral (5.94) is within the range of required precision $\epsilon_1(10^{-14})$.

For condition $t = \tau$ the integral (5.93) is in fact an integral of Weber and Schafheitlin and it is possible to calculate it analytically (A.10) [12].

Let us now consider the calculation of middle integrals

$$(5.95) \quad \tilde{I}_{\pm}(r, t, \tau) = \int_0^\infty T_{2,2n,\varphi,\varphi}(\tilde{r}, \tau) I_{\pm}(r, \tilde{r}, t, \tau) d\tilde{r}$$

We use asymptotical properties of confluent hypergeometric functions $\Phi(a, c; x)$ [10] and we have:

$$(5.96) \quad \begin{aligned} T_{2,2n,\varphi,\varphi}(\tilde{r}, \tau) &\rightarrow (1/\tilde{r}^{2n+5}) \\ \tilde{r} &\rightarrow \infty \end{aligned}$$

Hence, we substitute integral (5.95) for

$$(5.97) \quad \tilde{I}_{\pm}(r, t, \tau) = \int_0^{A_2} T_{2,2n,\varphi,\varphi}(\tilde{r}, \tau) I_{\pm}(r, \tilde{r}, t, \tau) d\tilde{r}$$

where $(0 < A_2 = 20 < \infty)$ and integration is really over the finite interval. For additional check let us increase value A_2 in 1.5 times and change the number of integration steps n_2 (from 201 to 301). We see that the difference in result values of integral (5.97) is within the range of required precision $\epsilon_2(10^{-11})$.

Confluent hypergeometric functions $\Phi(a, c; x)$ were computed with precision $\epsilon(10^{-15})$.

The outer integrals in (5.78), (5.79) are the integrals over finite interval $(0, t = 10)$. These integrals are computed with precision $\epsilon_3(10^{-5})$ and the number of steps of integration $n_3 = 101$. For additional check let us change the number of integration steps n_3 (from 101 to 201), and we see that the difference in result integral values is within the required precision $\epsilon_3(10^{-5})$. All integrals were computed by Simpson's method and $\epsilon_1(10^{-14}) < \epsilon_2(10^{-11}) < \epsilon_3(10^{-5})$.

$\vec{u}_2 = \vec{u}_1 - \vec{u}_2^*$ and is shown in FIG. 5.1.1 - 5.1.15. The vector field \vec{u}_2 at distances $r = 1, 2, 3, 5, 7$ is represented by the dotted curves in top diagrams. The comparison of $|\vec{u}_1|$ (dashed plots) and $|\vec{u}_2^*|$ (solid plots) in plane $\varphi = [0, \pi]$, at distances $0 \leq r \leq 50$ is represented in bottom diagrams. This comparison shows $|\vec{u}_2^*| \ll |\vec{u}_1|$ and is corresponding to the conclusion (4.89).

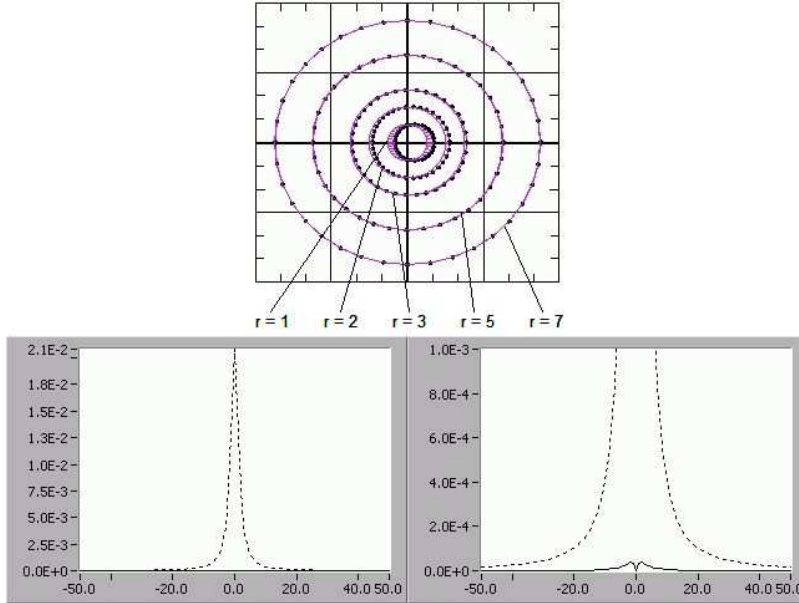
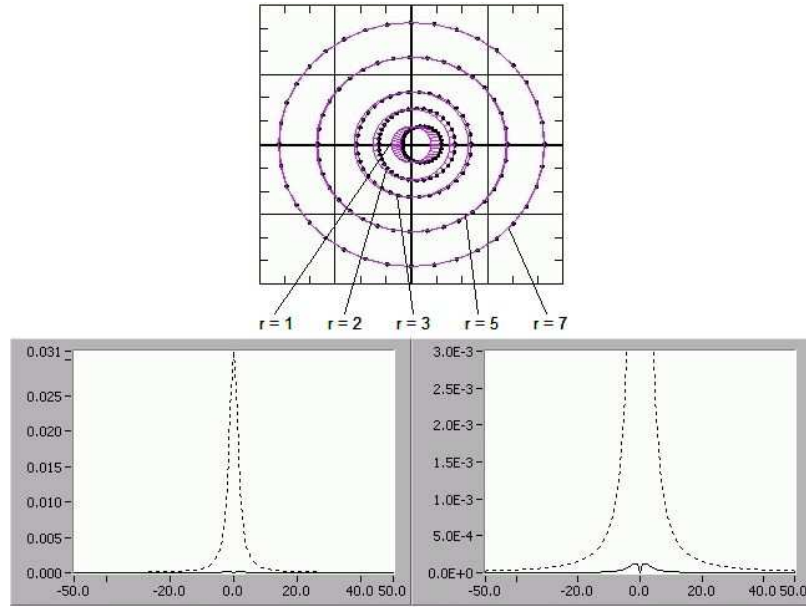
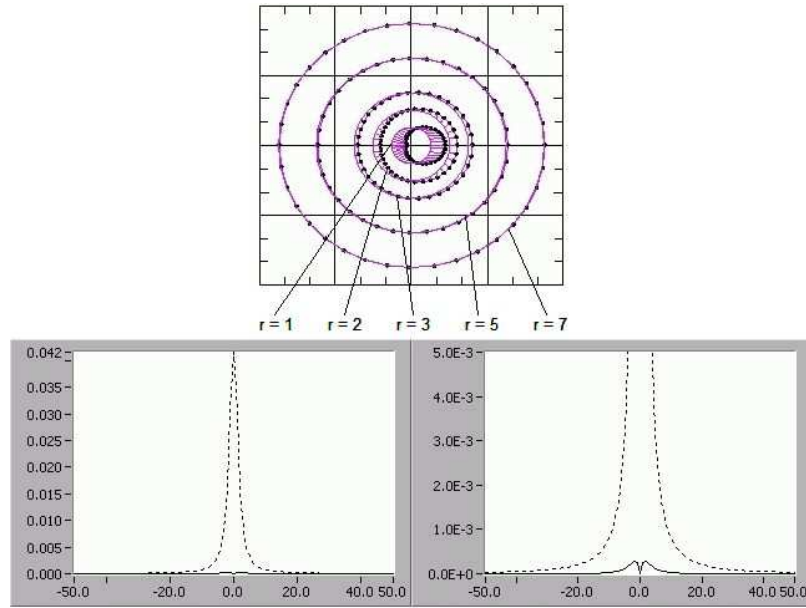
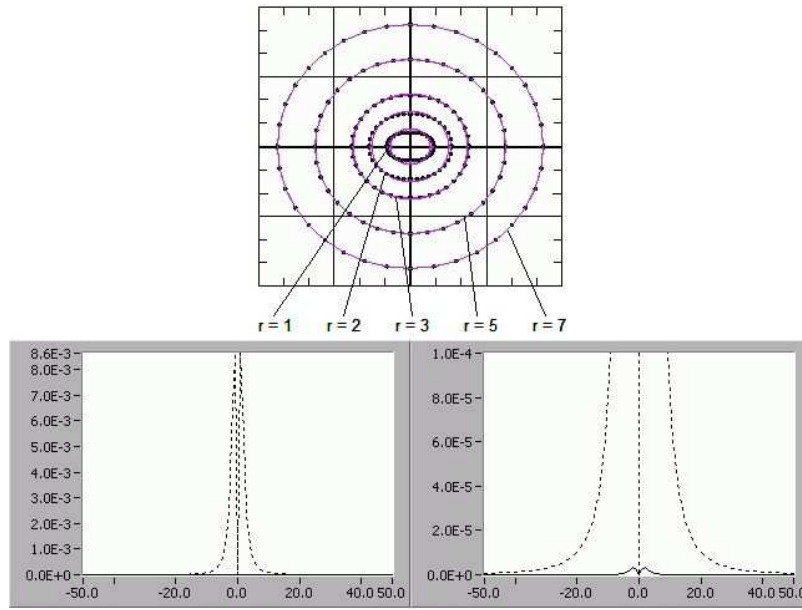
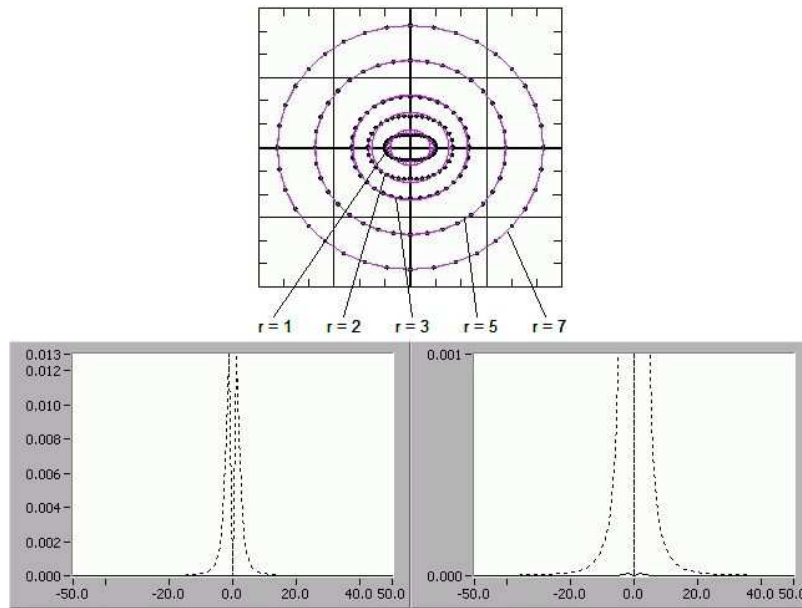
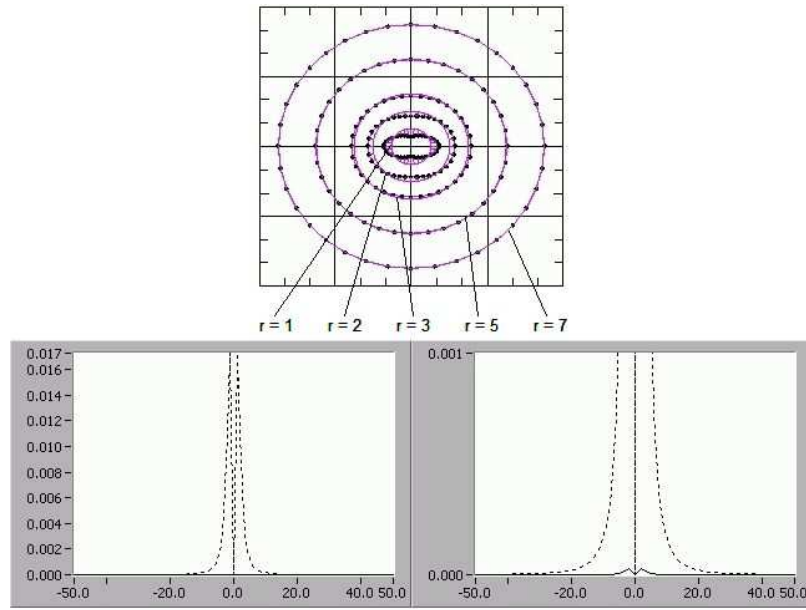
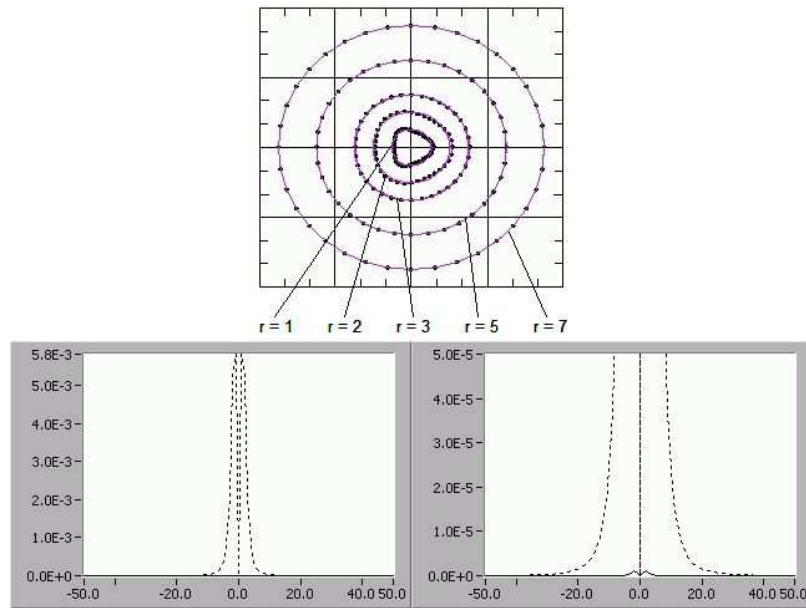
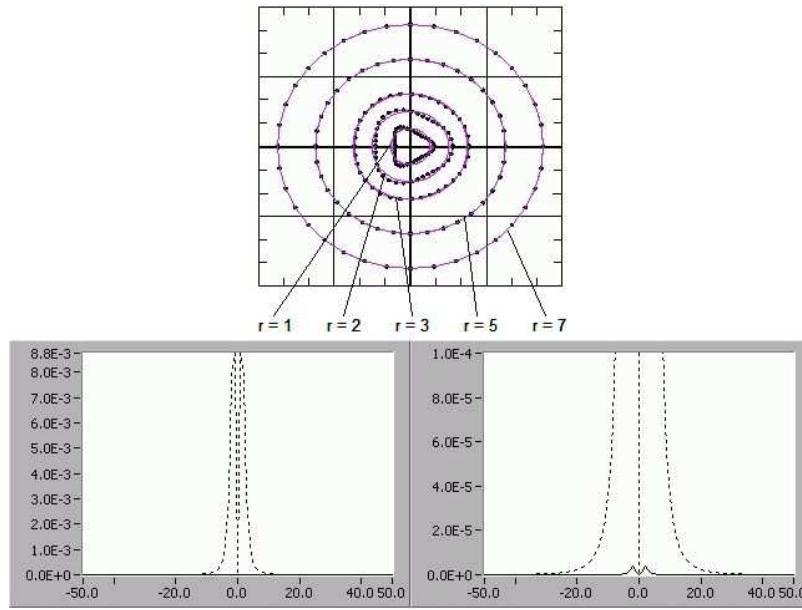
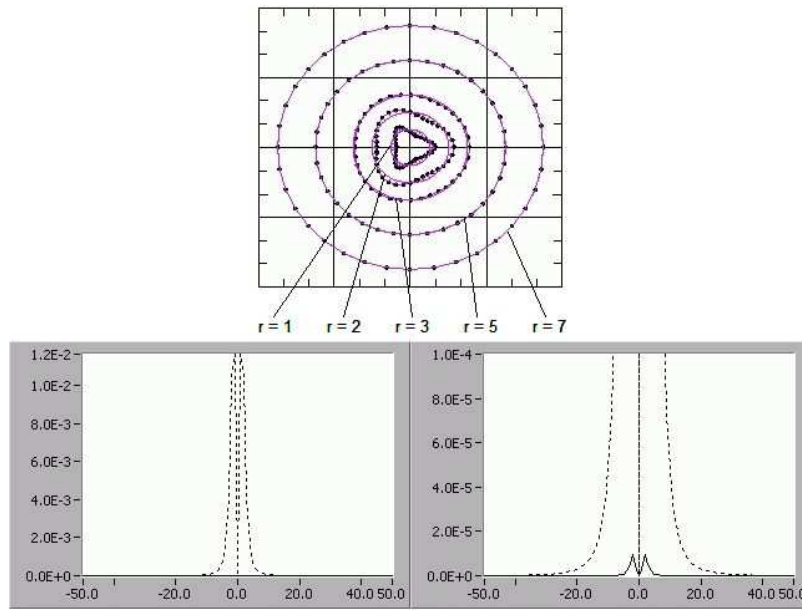


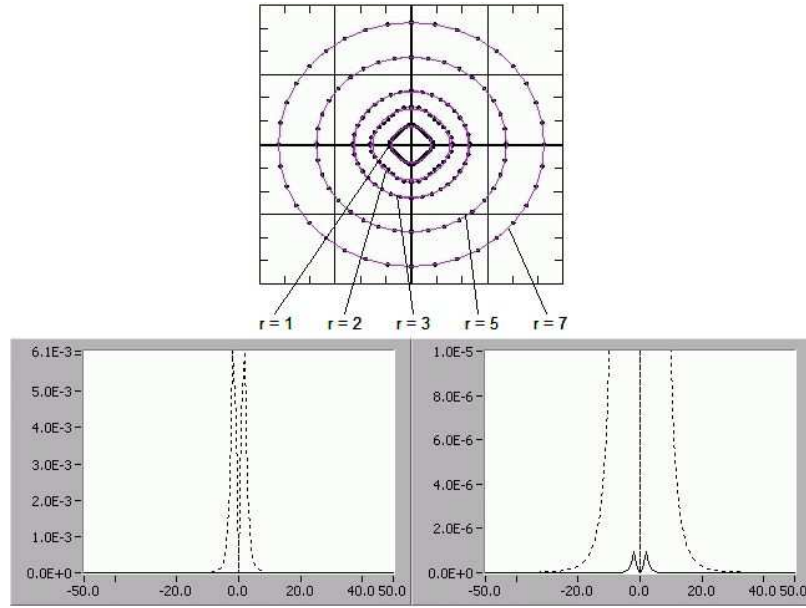
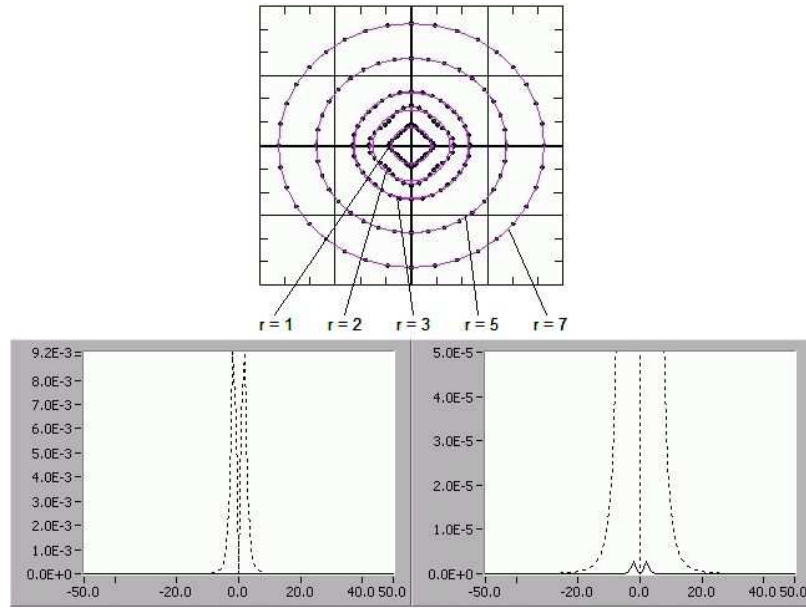
FIG.5.1.1. $n = 1, F_1 = 1, \nu = 1.5$

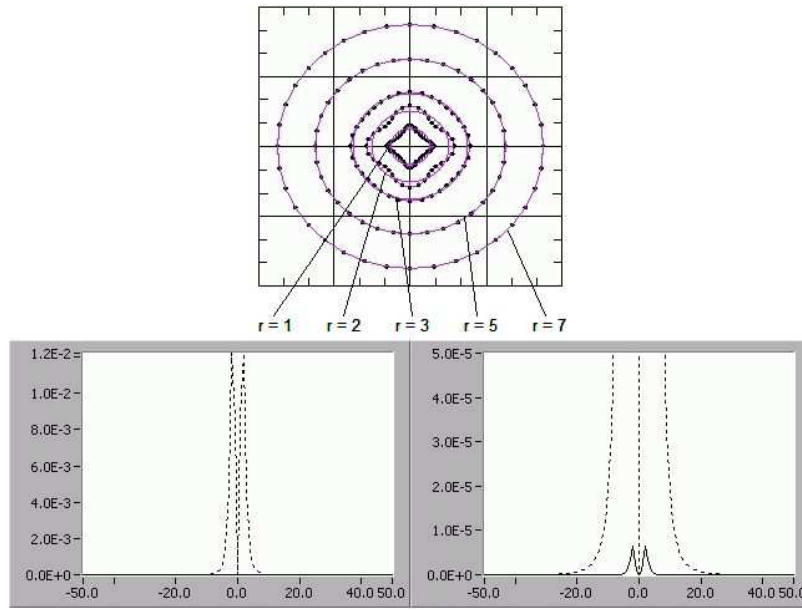
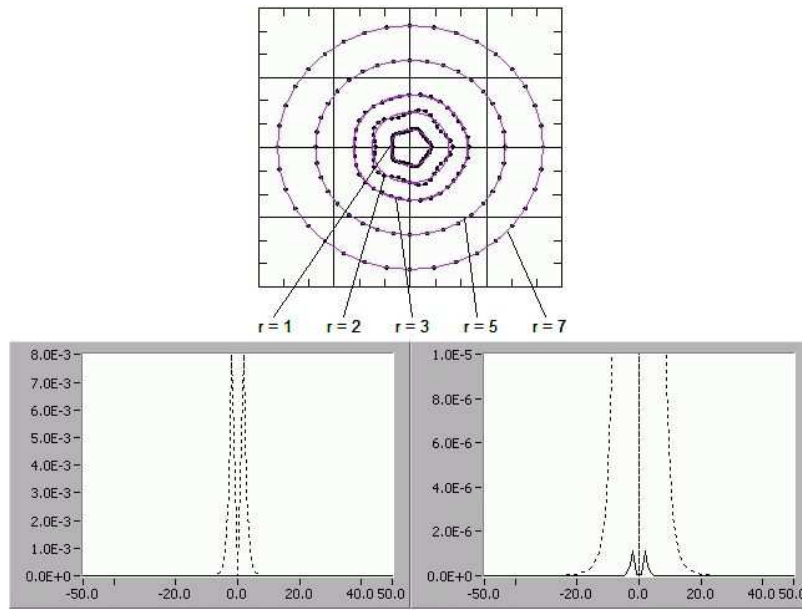
FIG.5.1.2. $n = 1$, $F_1 = 1$, $\nu = 1$ FIG.5.1.3. $n = 1$, $F_1 = 1$, $\nu = 0.75$

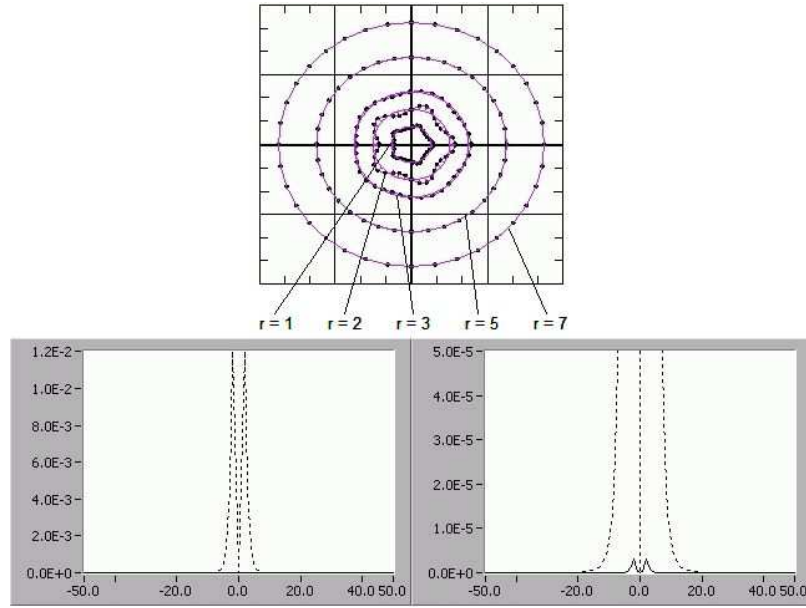
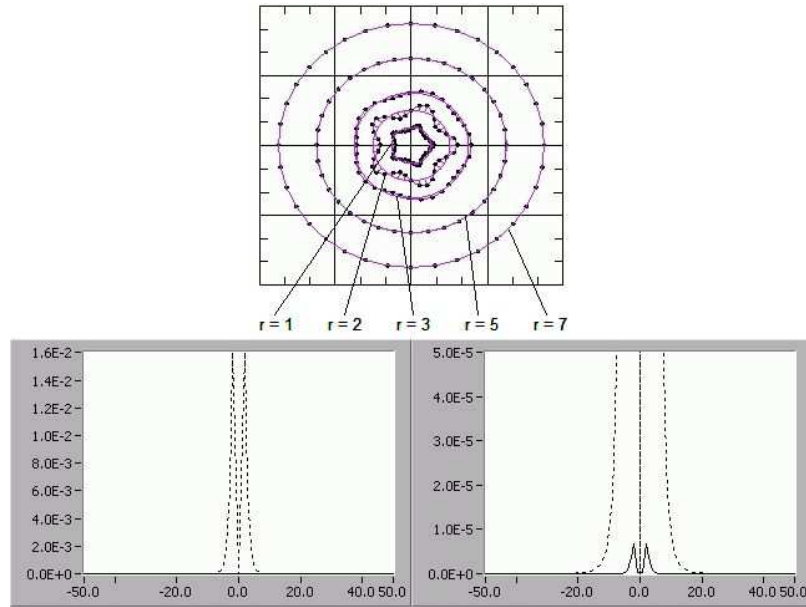
FIG.5.1.4. $n = 2$, $F_2 = 0.5$, $\nu = 1.5$ FIG.5.1.5. $n = 2$, $F_2 = 0.5$, $\nu = 1$

FIG.5.1.6. $n = 2$, $F_2 = 0.5$, $\nu = 0.75$ FIG.5.1.7. $n = 3$, $F_3 = 0.33$, $\nu = 1.5$

FIG.5.1.8. $n = 3$, $F_3 = 0.33$, $\nu = 1$ FIG.5.1.9. $n = 3$, $F_3 = 0.33$, $\nu = 0.75$

FIG.5.1.10. $n = 4$, $F_4 = 0.25$, $\nu = 1.5$ FIG.5.1.11. $n = 4$, $F_4 = 0.25$, $\nu = 1$

FIG.5.1.12. $n = 4$, $F_4 = 0.25$, $\nu = 0.75$ FIG.5.1.13. $n = 5$, $F_5 = 0.2$, $\nu = 1.5$

FIG.5.1.14. $n = 5$, $F_5 = 0.2$, $\nu = 1$ FIG.5.1.15. $n = 5$, $F_5 = 0.2$, $\nu = 0.75$

Appendix A.

The Fourier integral can be stated in the forms:

$$N = 1$$

$$(A.1) \quad U(\gamma) = F[u(x)] = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} u(x) e^{i\gamma x} dx \quad u(x) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} U(\gamma) e^{-i\gamma x} d\gamma$$

$$N = 2$$

$$(A.2) \quad \begin{aligned} U(\gamma_1, \gamma_2) &= F[u(x_1, x_2)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x_1, x_2) e^{i(\gamma_1 x_1 + \gamma_2 x_2)} dx_1 dx_2 \\ u(x_1, x_2) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\gamma_1, \gamma_2) e^{-i(\gamma_1 x_1 + \gamma_2 x_2)} d\gamma_1 d\gamma_2 \end{aligned}$$

$$N = 3$$

$$(A.3) \quad \begin{aligned} U(\gamma_1, \gamma_2, \gamma_3) &= F[u(x_1, x_2, x_3)] = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x_1, x_2, x_3) e^{i(\gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3)} dx_1 dx_2 dx_3 \\ u(x_1, x_2, x_3) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\gamma_1, \gamma_2, \gamma_3) e^{-i(\gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3)} d\gamma_1 d\gamma_2 d\gamma_3 \end{aligned}$$

The Laplace integral is usually stated in the following form:

$$(A.4) \quad U^{\otimes}(\eta) = L[u(t)] = \int_0^{\infty} u(t) e^{-\eta t} dt \quad u(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} U^{\otimes}(\eta) e^{\eta t} d\eta \quad c > c_0$$

$$(A.5) \quad L[u'(t)] = \eta U^{\otimes}(\eta) - u(0)$$

THE CONVOLUTION THEOREM A.1.

If integrals

$$U_1^{\otimes}(\eta) = \int_0^{\infty} u_1(t) e^{-\eta t} dt \quad U_2^{\otimes}(\eta) = \int_0^{\infty} u_2(t) e^{-\eta t} dt$$

absolutely converge by $\text{Re } \eta > \sigma_d$, then $U^{\otimes}(\eta) = U_1^{\otimes}(\eta) U_2^{\otimes}(\eta)$ is Laplace transform of

$$(A.6) \quad u(t) = \int_0^t u_1(t - \tau) u_2(\tau) d\tau$$

Useful *Laplace integral*:

$$(A.7) \quad L[e^{-\eta_k t}] = \int_0^\infty e^{-(\eta - \eta_k)t} dt = \frac{1}{(\eta - \eta_k)} \quad (Re \eta > \eta_k)$$

De Moivre's formulas:

$$(A.8) \quad \cos \varphi = \frac{1}{2}(e^{i\varphi} + e^{-i\varphi}) \quad , \quad \sin \varphi = \frac{1}{2i}(e^{i\varphi} - e^{-i\varphi})$$

Bessel function's integral representation:

$$(A.9) \quad J_n(z) = \frac{i^{-n}}{2\pi} \oint_C e^{iz \cos \theta + in\theta} d\theta \quad , \quad C \text{ is the unit circle around the origin.}$$

The discontinuous integral of Weber and Schafheitlin:

$$(A.10) \quad \int_0^\infty J_\mu(at) \cdot J_{\mu-1}(bt) dt = \begin{cases} b^{\mu-1}/a^\mu & (b < a) \\ 1/2b & (b = a) \\ 0 & (b > a) \end{cases}$$

$$(A.11) \quad \int_0^\infty J_\mu(\alpha t) e^{-\gamma^2 t^2} t^{\mu+1} dt = \frac{\alpha^\mu}{(2\gamma^2)^{\mu+1}} e^{-\frac{\alpha^2}{4\gamma^2}}, \quad Re \mu > -1, Re \gamma^2 > 0.$$

$$(A.12) \quad \int_0^\infty J_\mu(\alpha t) e^{-\gamma^2 t^2} t^{\rho-1} dt = \frac{\gamma^{-\rho}}{2 \cdot \Gamma(\mu+1)} \cdot \Gamma\left(\frac{\mu+\rho}{2}\right) \cdot \left(\frac{\alpha}{2\gamma}\right)^\mu \cdot \Phi\left(\frac{\mu+\rho}{2}, \mu+1; -\frac{\alpha^2}{4\gamma^2}\right),$$

$Re \gamma^2 > 0, Re(\mu + \rho) > 0.$

$$(A.13) \quad \frac{d}{dy}[y^a \cdot \Phi(a, c; -\beta y)] = a \cdot y^{a-1} \cdot \Phi(a+1, c; -\beta y)$$

$$(A.14) \quad \frac{d}{dy}[y^{c-1} \cdot \Phi(a, c; -\beta y)] = (c-1) \cdot y^{c-2} \cdot \Phi(a, c-1; -\beta y)$$

Formula describing connection between the contiguous confluent hypergeometric functions:

$$(A.15) \quad c \cdot \Phi - c \cdot \Phi(a-) - x \cdot c \cdot \Phi(c+) = 0$$

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